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UNCOVERING TRANSFORMATIVE EXPERIENCES: A CASE

STUDY OF THE TRANSFORMATIONS MADE BY ONE TEACHER

IN A MATHEMATICS PROFESSIONAL DEVELOPMENT

PROGRAM

by

Rachelle M. Orsak

A thesis submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Master of Arts

Department of Mathematics Education

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GRADUATE COMMITTEE APPROVAL

of a thesis submitted by

Rachelle M. Orsak

This thesis has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

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BRIGHAM YOUNG UNIVERSITY

As chair of the candidate's graduate committee, I have read the thesis of Rachelle Orsak
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consistent and acceptable and fulfill university and department style requirements; (2) its
illustrative materials including figures, tables, and charts are in place; and (3) the final
manuscript is satisfactory to the graduate committee and is ready for submission to the
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ABSTRACT

UNCOVERING TRANSFORMATIVE EXPERIENCES: A CASE STUDY OF THE
TRANSFORMATIONS MADE BY ONE TEACHER IN A MATHEMATICS
PROFESSIONAL DEVELOPMENT PROGRAM

Rachelle M. Orsak

Department of Mathematics Education

Master of Arts

Effective professional development is vital for improving mathematics teaching
(National Council of Teachers of Mathematics [NCTM], 2007), so studying effective
professional development programs is important to the field of mathematics education.
This case study presents findings on one teacher, Rebecca, and her experiences in a five-
semester mathematics professional development for elementary teachers. The
participants in this professional development engaged in collaborative problem solving of
challenging mathematical tasks over extended periods of time. I used qualitative research
methods based on grounded theory methodology (Charmaz, 2006) to analyze Rebecca's
entrance and exit surveys, video data of Rebecca's individual interviews, and video data
of Rebecca and her collaborative group problem solving in the professional development.
Analysis shows that through the professional development program, Rebecca had

transformative experiences which led to significant changes in her perspectives and practices. This case study contributes to the field of mathematics education a better understanding of the transformations teachers can experience through professional development as well as some particular conditions for professional development programs to be successful in offering teachers opportunities for transformative experiences.

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This thesis is dedicated to Trey and Conner, because I love them.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

Teachers' knowledge and pedagogical skills are extremely influential with respect to the opportunities students have for learning. Clearly, the training teachers receive before entering the school system as professional educators is important for their success (Darling-Hammond, 2000), but in the long run, even more critical is the continuation of teacher education throughout an entire teaching career. As Barth (1980) wrote, "Probably nothing within a school has more impact on students in terms of skill development, self-confidence, or classroom behavior than the personal and professional growth of their teachers. . . When teachers stop growing, so do their students" (p. 147). Teachers' professional education is necessary for good student learning, and researchers have noted that any improvement in education must begin with teacher learning (Borko, 2004; Sykes, 1996). Darling-Hammond (1998) further emphasized the centrality of teacher learning for improving education when she wrote "At its root, achieving high levels of student understanding requires immensely skillful teaching—and schools that are organized to support teachers' continuous learning" (p. 7). Hence, understanding the mechanisms by which teachers actually learn mathematics and grow professionally is important for understanding how to achieve excellence in education.

Continued professional development is necessary for all teachers. In particular, teachers of mathematics need opportunities to build conceptual knowledge (National Council of Teachers of Mathematics [NCTM], 2007) if they are to help students learn mathematics concepts. Additionally, most teachers have experienced very traditional teaching styles throughout their own schooling and thus may have traditional perspectives of mathematics learning and teaching (Schifter, 1996). Such teachers may

need special experiences to help them "shift their perspectives about teaching" (NCTM,

2007, p. 5) so they can be open to new possibilities to better enable their students to

understand and use mathematics concepts. Hence, professional development, especially

how it leads teachers to improve, is a significant topic for research in mathematics

education.

CHAPTER 2: THEORETICAL PERSPECTIVE

It has been well established that learners construct their own knowledge through experience and that it is based on their prior understandings and organization of knowledge (Cobb, Wood & Yackel, 1993; Cobb, Yackel & Wood, 1992; Simon & Schifter, 1991; von Glassersfeld, 1983). Individuals only add to or modify their organization of knowledge if they encounter some inadequacy or perceived conflict between their current cognitive structures and their current experience (von Glassersfeld, 1983). By this reasoning, learners cannot have knowledge transmitted by a teacher, but instead they learn through experiences that provide a cognitive need to construct or reconstruct knowledge. I see teachers as learners of pedagogy and content, and as such, they cannot simply have knowledge for teaching transmitted to them from some outside source. To learn how to teach well, teachers must have experiences that lead them to construct or reconstruct understandings of what counts as mathematical activity, how students best learn mathematics, and what comprises exemplary mathematics teaching.

Researchers have shown that teachers' perspectives on mathematics, learning, and teaching profoundly influence their teaching practices (Ho & Hedberg, 2005; Simon & Schifter, 1991). Simon and Schifter (1991) discussed the influence of teachers' perspectives on their practices and concluded that "how teachers think about mathematics learning is a key determinant of how they teach" (p. 329). Since teaching practices affect student learning, it is important to examine teachers' perspectives and how they might change those perspectives. When studying actual teaching practices is not feasible, analyzing teachers' perspectives can yield important results.

In response to perceived educational problems such as low test scores, the public is often quick to blame teachers (Boaler, 2008). It seems that many view teachers as simple practitioners who need to be motivated or forced to improve their practices; however, I choose to view teachers as professionals who strive to continually learn and improve (Loucks-Horsley, Hewson, Love, & Stiles, 1998). By believing in teachers' desires to teach in ways that provide their students with, in their opinions, the best possible opportunities to learn mathematics, I feel I am afforded more insights into the meaning of their actions than if I were to continually doubt their intentions.

Just as all children can learn important mathematics (NCTM, 2000), I believe that all teachers can learn to teach mathematics effectively. Of course, not all children learn important mathematics, at times because of lack of opportunity (whether it be lack of classes, lack of resources, or lack of quality teaching) but at other times because they choose to learn other things they find more important. Teachers' learning, too, can be affected by lack of opportunity or by personal choices. It is critical that teachers are given opportunities for constructing good teaching practices not only through well-designed preservice education but especially through thoughtful professional development. Additionally, teachers have personal agency (Walter & Gerson, 2007) and can choose what they feel are the important components of good practice and can choose to improve their practices accordingly. Although carefully planned professional development programs are imperative for improving education, teachers' individual choices shape their unique understandings and practices.

CHAPTER 3: LITERATURE REVIEW

Research has shown that many teachers' understandings of what mathematics and mathematics learning are "may be incompatible with current research and reform efforts in the field" (Senger, 1999, p. 200). Teachers may not have the mathematical or pedagogical knowledge needed for teaching mathematics well. All teachers generally need knowledge of subject matter, students, and learning (Borko, 2004; Darling-Hammond, 1998). Thus, mathematics teachers need to understand what teaching and learning mathematics entail. From a constructivist perspective, "teaching mathematics is to be understood as providing students with the opportunity and the stimulation to construct powerful mathematical ideas for themselves" (Simon & Schifter, 1991, p. 310). In order to provide such opportunities for students, mathematics teachers need deep knowledge of content, representations, and mathematical contexts (Darling-Hammond, 2000; Simon & Schifter, 1991), or as Borko (2004) described, teachers need "rich and flexible" mathematical knowledge (p. 5). Darling-Hammond (2000) asserted that, most importantly, teachers need to know how to "see beyond one's own perspective, to put oneself in the shoes of the learner and to understand the meaning of that experience in terms of learning" (p. 170).

Teachers construct their own understandings of mathematics, learning, and teaching. Teachers have their own unique sets of prior experiences that shape their current practices and future learning (Ball, 1996; Cobb, Yackel, & Wood, 1992; Simon & Schifter, 1993; von Glassersfeld, 1983). Consequently, teachers may learn best by participating in worthwhile experiences that help them to construct understandings. Such experiences may include "studying, doing and reflecting . . . collaborating with other

teachers . . . looking closely at students and their work, and . . . Teachers can have especially powerful experiences learning about mathematics, learning about learning, and learning about teaching when they engage as mathematics learners themselves (Ball, 1996; Borko, 2004; Graven, 2004; Leikin, 2004; Sykes, 1996; Simon & Schifter, 1993, 1991; Walter & Gerson, 2007). Walter and Gerson (2007) elaborated, "When teachers focus on their *own* mathematics learning within supportive professional development communities they are cognizant of pedagogical contexts that either constrain or offer affordances to *them* as learners personally building mathematics and questions related to teaching for understanding emerge" (p. 205).

Traditionally, professional development experiences such as inservice classes and workshops have been employed. However, such experiences have been judged as "sorely inadequate" for helping teachers construct viable understandings of mathematics or develop student-centered mathematics teaching practices (Sykes, 1996, p. 465; Borko, 2004; Loucks-Horsley et al., 1998; Ball, 1996). Traditional professional development has focused on transmitting knowledge or training for skills (Ball, 1996; Loucks-Horsely et al., 1998; Sykes, 1996). Ball (1996) described the main shortcoming of traditional professional development as a lack of "critical discussion," without which professional developers "rarely challenge teachers' assumptions or intentionally provoke disequilibrium or conflict" (p. 505). Without disequilibrium or conflict, teachers do not have cause to reorganize their existing cognitive structures. A lack of conflict does not promote the construction of new knowledge or practices because there is not a cognitive need to do so (von Glassersfeld, 1983). In other words, traditional professional development programs have simply "ignore[d] how teachers learn" (Borko, 2004, p. 3).

Traditional teacher preparation and professional development programs have been shown to be better than no training at all (Darling-Hammond, 2000), but the shortcomings offer an undeniable need for improvement.

Due to the limitations associated with traditional professional development and in response to recent calls for reforming mathematics education (NCTM, 2000), many have advocated reforming professional development "in ways that parallel the teaching reforms themselves" (Sykes, 1996, p. 466; see also Ball, 1996; Borko, 2004; Darling-Hammond, 2000; Graven, 2004; Loucks-Horsely et al., 1998; Schifter, 1996; Simon & Schifter, 1991; Walter & Gerson, 2007; Zaslavsky & Leikin, 2004). This reform includes shifting from the transmission or training model to one that is learner-centered, or centered on teachers as active learners of content and pedagogy (Zaslavsky & Leikin, 2004). Ball (1996) described good professional development as "critique and inquiry toward practice" which involves shifting from implementation to "generation of new knowledge" (p. 506). Engaging teachers as learners of mathematics is a powerful way to generate teachers' knowledge of content and pedagogy, especially under particular circumstances that elicit, in learners, conceptually important mathematics (Ball, 1996; Borko, 2004; Graven, 2004; Leikin, 2004; Simon & Schifter, 1993, 1991; Sykes, 1996; Walter & Gerson, 2007;). Such approaches to professional development will not provide teachers with a "recipe" for changing their teaching practices (Schifter, 1996, p. 496), but can instead promote teachers to develop new conceptions of what mathematics is, how mathematics should be learned, and consequently, how mathematics can be taught effectively.

It is important to note that although good professional development experiences may provide teachers with opportunities to enlarge their perspectives of mathematics, learning, and teaching, such experiences may also leave teachers wondering how they might implement their new ideas (Schifter, 1996; Simon & Schifter, 1991). Indeed, teachers may become frustrated by the slow progress in actually changing their practices. Schifter (1996) noted, "Teachers who begin this process expecting to develop a finished repertoire of behaviors that, once achieved, will become routine will be disappointed. Teaching this way is necessarily disruptive of routine, if for no other reason than that students will continually surprise us with their own discoveries" (p. 499). Hence the goal of professional development should not be for teachers to leave equipped with new, perfect practices. Instead, professional development must offer teachers opportunities for revising their perspectives which, in turn, will provide a beginning or encouraging waypoint on the journey of constructing better teaching practices.

Many have studied and described the processes through which teachers construct new teaching practices. Findings have established that change takes both time and effort (Ball, 1996; Borko, 2004; Cohen & Ball, 1990; Desimone, Porter, Garet, Yoon, & Birman, 2002; Schifter, 1996; Zaslavsky, & Leikin, 2004). In order to change practices, teachers must change their "conceptions of mathematics and learning" (Simon & Schifter, 1991, p. 312; see also Ball, 1996; Zaslavsky & Leikin, 2004). If teachers become more confident in their own ability to learn mathematics, they become empowered to be better able to change in ways that lead to more competent teaching (Graven, 2004). Reflection is a critical feature of the change process. Teachers are able to revise their assumptions as they have experiences that cause them to reflect on those

assumptions (Ball, 1996; Borko, 2004; Senger, 1999; Zaslavsky & Leikin, 2004). Senger (1999) gave a framework for the change process, which includes questioning, reflection, experimentation in verbalization and in practice, and changes in verbalization and in practice (see fig. 2, p. 211). Senger clarified that the change process is "recursive," involving a "folding back between different ways of experimenting rather than a linear movement in stages" (p. 210). Others have focused on the distinction between behavioral and epistemological changes (Gill, Ashton, & Algina, 2004; Spillane & Zeuli, 1999). These authors showed that behavioral or surface level changes, such as using group work in the classroom, are easier to implement than more fundamental changes in epistemology. Teachers' personal agency has also been shown to be a critical component of the change process (Simon & Schifter, 1991; Walter & Gerson, 2007). Lasting changes in teaching practices are possible. The processes which influence those changes are long-term and complex.

While many have offered descriptions, in general, of some of the complex processes teachers experience in order to change their perspectives and practices, researchers have argued for the need to take a closer look at the actual mechanisms that lead to change (Borko, 2004; Gill et al., 2004; Senger, 1999; Simon & Schifter, 1993). Senger (1999) wrote, "Understanding the internal processes individual teachers go through as they change is critical to researchers and educators during this vital era of reform" (p. 199). Simon and Schifter (1993) also emphasized the need to look at teacher change in finer detail when they exhorted researchers to "more closely examine change in teachers' conceptions of mathematics, and of learning and teaching" (p. 337).

Responding to the need for a closer examination of the processes through which teachers change their perspectives and practices, I studied such changes made by one teacher in a particular professional development program. Schifter (1996) described the process of constructing effective practices as requiring "a qualitative transformation of virtually every aspect of mathematics teaching" (p. 497). Senger (1999) found that these transformations are part of a "complex and thoughtful process" (p. 214). This thesis is a study of the complex process of one teacher changing, or transforming, her perspectives and practices in a professional development with special emphasis on the individual *transformative experiences* that made up the entire process. My working definition of a transformative experience is an observable occurrence that significantly influences one's perspective on mathematics, learning, or teaching.

Purpose

The purpose of this study is to identify and carefully characterize the transformative experiences of one teacher in a particular professional development program. In order to study transformative experiences, I identified and studied the actual transformations, or changes in perspectives and practices, that this teacher achieved during the professional development. This case study provides further insight into the change process of teachers constructing new understandings and practices in a mathematics professional development program.

CHAPTER 4: METHODOLOGY

Setting

The data for this study come from a long-term mathematics professional development program for practicing elementary teachers designed by and under the direction of Dr. Janet Walter. The professional development program was a broader study conducted at a large, private university in the Western United States. A local school district requested the professional development. One or two teachers from each elementary school in the district participated, for a total of twenty-five participants. The group met weekly for three-hour sessions over five semesters. An important aspect of the program design was that the teachers participated in the professional development program after school during three academic years, not during the summer. Thus, the teachers were continually going back and forth between the learning experiences of the professional development and the learning-in-action that occurred in their own classrooms.

Each week, participants worked in collaborative groups on conceptually-rich, open-response tasks that spanned multiple class sessions. The practicing teachers in this professional development program were engaged as learners of mathematics; the participants did not simply learn more about their elementary curricula, but were required to collaboratively solve mathematically challenging tasks in algebra, geometry, trigonometry, calculus, and statistics. Groups regularly gave informal presentations to the entire class to explain their solutions or works-in-progress. Also, individuals would give periodic, more formal presentations on lessons or units they developed and implemented in their elementary classrooms. These presented lessons and units were always based on

the mathematical content of the most recent mathematics tasks on which the teachers-as-learners collaborated in the professional development classes.

Protocols for Collecting Data

During each weekly session, participant interactions were videotaped from the moment the first participant entered the classroom until the last participant left. The evening class sessions were scheduled for three hours, but due to some participants arriving early and many staying late to discuss ideas, four or five hours of videotape were recorded each week. One video camera was running during class sessions and microphones were set up at tables to pick up participants' conversations. For each task, one collaborative group was selected as a focus group and a camera was dedicated to videotape that group. Over time, all collaborative groups were recorded. Participants' group presentations of solutions or individual presentations of lesson plans were also captured on video. Camera work was done by undergraduate and graduate student research assistants in mathematics education.

Individual participant interviews were videotaped at the end of the fourth and fifth semesters. Interviews were based on a flexible protocol (Appendix A) and were conducted by one of the principal researchers speaking with one participant at a time. Undergraduate and graduate research assistants videotaped each interview. Interviews lasted approximately fifteen minutes each. Interview questions which relate to this research on transformative experiences include the following:

- What have you learned this semester?

- What has changed about how you view yourself as a participant in class?

- How has this professional development influenced what you teach?

- How has this professional development influenced how you teach?

- How has this professional development influenced how you view mathematics learning?

- How has this professional development influenced what mathematics your students do?

- How have your students benefited from your participation in the entire program?

Data

For this research, individual participant interview videos from one participant, Rebecca, are the primary sources of data. Rebecca's self-reports yield insight into her transformative experiences in the professional development. Recall that transformative experiences are defined as observable occurrences that significantly influence an individual's perspectives on mathematics, learning, or teaching. Because Rebecca might not always be able to articulate her perspectives or identify transformative experiences in an interview setting, the interview data was triangulated by an analysis of classroom video of Rebecca and her collaborative group of teachers building mathematics through problem solving in one specific task: the *Placenticeras* task. The *Placenticeras* task was chosen for analysis because the collaborative interactions of the group were representative of the professional development sessions but also because the task was particularly compelling, especially for Rebecca. Surveys from the beginning and end of the professional development program were also used to provide background detail and enhance the analysis offered here. Copies of Rebecca's mathematical work (notes, write-ups, tests, etc.) were also available to supplement the video record.

Participants

The focus group for the *Placenticeras* task consists of six teachers: Rebecca, Brenda, Christine, Lyn, Lorri, and Amber. These participants worked together throughout the entire third semester of the professional development. Due to absences, Lyn, Lorri, and Amber played minor roles in the gathering and analyzing of data. Interactions and collaborative efforts between Rebecca, Brenda, and Christine were extensively analyzed.

At the beginning of the professional development, Rebecca was a first-year, fifth-grade teacher. Rebecca displayed infectious enthusiasm in nearly everything she did, and she felt that she knew how to make mathematics exciting for her students; however, she reported that she did not always feel confident in her own mathematical ability or her ability to teach mathematics effectively to her students. When asked at the beginning of the program what she disliked most about mathematics, Rebecca answered, "How I was taught math," because "it seemed too complicated to ever really be enjoyable."

Brenda began the professional development program after teaching third and sixth grades for just over three years. Brenda had an outgoing personality and was a major contributor to group and whole-class discourse. However, near the beginning of the program she said, "I don't like having to explain why," and described herself as having a "lack of conceptual knowledge."

Christine began the program with five years of experience teaching fourth grade. Of the six focus group members, Christine had the most extensive upper-level mathematics background, having taken mathematics classes through calculus as an undergraduate. In describing what she liked about mathematics, Christine wrote, "I love the puzzle/discovery part of it. I tend to view math as a puzzle that I am trying to solve. I

like these kinds of things." Christine described herself as confident in her own

mathematical ideas, and she was often willing to take charge in group situations.

<div align="center">The Task</div>

The *Placenticeras* task[1] was given to participants during the third semester of the

program; this semester was focused on trigonometry content. In the *Placenticeras* task,

participants were asked to work collaboratively to mathematically model the spiral of a

fossilized shell. (See Figure 1)

This task is about a spiral shell. The shell is a fossil *Placenticeras*, an ammonite that fell
to the bottom of a shallow sea 170 million years ago near what is now Glendive,
Montana, and was buried in a mudslide. Several people can join forces to build a
solution together. The actual shell size is half the size of the shell in the photocopy. The
larger size of the photocopy makes the shell structure more visible.

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[1] The Placenticeras task was originally developed in 1991 for university honors students by Bob Speiser,
BYU, and was adapted for participants in the professional development trigonometry course.

1. On your photocopy of the shell, locate the center of the spiral, and then draw a ray from this center, pointing in any direction you like.

2. Having chosen a center and a ray, we can now begin to describe the spiral of the shell. Use the photocopy and a ruler to collect data for *r* as a function of *theta*. Based on the information you have gathered, what can you say about *r* as a function of *theta*?"

Figure 1. A copy of the *Placenticeras* task given to participants in the professional development program.

Individuals' Roles

The principal investigator in the larger study was the instructor in the professional development course. She also designed the data collection, organized the structure of the professional development, planned and implemented tasks, orchestrated the collection of data, obtained IRB approval, and hired undergraduate research assistants. My role, as a member of the research team, was to analyze extant video data to characterize Rebecca's personal transformations and her transformative experiences. In the context of the professional development, the participants' roles were to be learners of mathematics, to build personal expertise in teaching mathematics, and to prepare to become numeracy (or mathematics) specialists at their own schools. Some teachers were also working toward level-two state certification; others were taking additional non-mathematics courses during the summer to fulfill requirements for master's degrees.

Methods of Analysis

The analysis of Rebecca's transformative experiences in this professional development is based on Rebecca's interview videos as well as excerpts taken from classroom videos of participants' interactions as they engaged in solving the *Placenticeras* task. Analysis techniques were consistent with qualitative methods in education research for analyzing video data (Powell, Francisco, & Maher, 2003), which include seven, non-linear phases:

1. viewing the video

2. describing the video

3. identifying critical events

4. transcribing

5. coding

6. constructing a storyline

7. composing narrative

Powell et al. (2003) stressed that all seven phases are "interacting" and "non-linear" (p. 413), so as many iterations of the techniques were conducted, the outcomes of each phase guided my work on the other phases.

Viewing the video was a critical part of this qualitative study. Initially, intensive viewing of the videos allowed a strong familiarity with these data. Repeated viewings guided descriptions and codes and continued to shape and reshape analysis throughout the entire writing process.

Further, through creating descriptions, a much deeper understanding of the data emerged than was provided by simply watching the video. In particular, the act of writing descriptions brought forward emerging themes that guided the coding. Descriptions were also helpful in initially looking for overall patterns.

Two undergraduate research assistants and I transcribed the video using Transana software (Fassnacht & Woods, 2008). Transcripts were annotated to include body language and gestures, with special attention given to the participants' actions during mathematics problem solving. Annotations were written within square brackets. Transcripts were then verified by members of the research team and time codes were

inserted using Transana. Time codes were inserted at the beginning of each speaker turn for the classroom video. Since Rebecca's interviews consisted of many long, uninterrupted segments of her speaking, time codes were inserted at approximately each sentence. Transcribing the class sessions facilitated understanding the dialogue in more precise detail than only viewing or describing could do. Transcribing interviews gave a more accurate view of what participants had said than descriptions could afford, and aided in making coding more efficient, complete, and grounded in the actions, utterances and self-reported experiences of the participants.

Coding was an important component in my analysis of transformative experiences. In a grounded theory approach to data analysis, codes are not predetermined, but are developed as themes emerge through viewing, describing, and transcribing the video data (Charmaz, 2006).

Rebecca's two interviews were the first data to be coded. Interview transcripts were examined and initial emergent codes recorded, resulting in over 120 initial codes. After two complete passes of open coding interview transcripts, the emergent codes were focused into a more manageable collection of 47 broader codes, referred to as the Interview codes. For example, initial emic codes such as "acknowledging weakness," "realizing there's more to go," "needing to improve," "expressing shortcomings," "feeling inadequate," and "noting inadequacy" were all combined into one broader code, "noting inadequacy." Next, Transana was used to code each individual time-coded segment of Rebecca's interview utterances with the broader codes from the collection. As important ideas emerged, I added to and refined the collection of Interview codes. (For a complete list of the collection of Interview codes, see Appendix B.)

Rebecca's background survey was then searched for pertinent information. Basic facts about her background, such as years of teaching experience and grade level, were recorded. Special attention was given to any responses that illustrated her perspectives on mathematics, teaching, and learning or her teaching practices upon entering the professional development. A summary of Rebecca's responses is given in Chapter 5.

In addition to Interview codes, *Placenticeras* task codes were developed through repeated viewings of the five hours of classroom video. Codes were created for important mathematical terms in order to keep track of the mathematical language the participants used throughout the task. Codes were also created for behaviors such as questioning, collaborating, predicting, etc. (For a complete list of *Placenticeras* task codes, see Appendix B). Within Transana, most classroom video transcript lines were coded, but unlike the interviews, I did not code every single speaker turn. I made my best judgment as to what lines were irrelevant or off topic and did not code those segments. Each pertinent speaker turn was coded with every relevant code from the *Placenticeras* task codes. I also coded classroom transcript lines with Interview codes. This was done to not only analyze the *Placenticeras* task and the mathematical work of the focus group, but to also connect the analysis of the interviews back to the task. Through carefully watching and layered coding of the videos, I pieced together the storyline of the mathematical work of the focus group, which is discussed in Chapter 5.

Once coding was complete, I sorted codes into core theoretical categories (Charmaz, 2006) and performed various frequency searches within Transana. Sorting and searching the codes brought forward many important relationships and provided areas in

which to focus the analysis. The details of the sorting and searching can be found in

Chapter 5, with results presented and discussed in Chapters 6 and 7.

CHAPTER 5: DATA AND ANALYSIS

The data and analysis are presented in this chapter. The reader should be advised that this data could be the basis for a myriad of interesting and important analyses, but the various other directions will take background to the focus of interest, transformative experiences. First, I describe Rebecca's background as well as the mathematical background of the entire focus group. Next, I present a narrative of the mathematical work completed by the focus group on the *Placenticeras* task. I then give brief descriptions of Rebecca's responses in her two interviews, followed by an overview of Rebecca's responses to her exit survey. Lastly, I discuss the development and emergence of core theoretical categories and further analysis techniques.

Rebecca's Responses to the Initial Survey

All participants in the broader study completed two surveys, the first near the beginning of the first semester of the professional development and the second at the end of the professional development project. In the survey, participants were asked to give various facts about their teaching experience and mathematics background (years teaching, mathematics courses taken, etc.) as well as describe their perspectives on a few issues of learning and teaching mathematics (Appendix C contains a copy of Rebecca's initial survey). I gathered information from Rebecca's initial survey to describe her teaching and mathematical backgrounds and have divided it into two separate sections: the first on Rebecca's teaching background and the next on her mathematics background.

Rebecca's Teaching Background and Perspectives

Rebecca began the professional development while she was in her first year of teaching. Throughout the entire program, Rebecca taught fifth grade at the same

elementary school. When asked in the initial survey for three necessary qualities of an excellent mathematics teacher, Rebecca answered, "Experience based knowledge of math and how students best learn it, confidence, and creativity (mixed with a touch of humor)." Of these areas, Rebecca felt strongest in being creative and knowing how students best learn mathematics. She felt that she could keep mathematics exciting for her students and that she was aware of each student's engagement. Rebecca felt weakest in her knowledge of mathematics. In response to the survey question, "Which of the qualities you listed above, do you feel is your weakest? Please explain," Rebecca wrote,

> Knowing math inside and out well enough to be able to break it down for my slower students, and explain the hows and whys to my higher students.

Rebecca felt too inadequate in her mathematical knowledge to present the information well or explain it effectively. To be able to teach mathematics to her students, Rebecca felt that she needed to learn more.

In the initial survey, Rebecca described her optimum mathematics classroom as a "scare-free environment" where mathematics was integrated with other subjects, the teacher was enthusiastic, mathematics was fun, and students were allowed to investigate their own answers and methods of solving problems. Rebecca reported that her own classroom did not match her imagined, optimum classroom, but she tried to show why certain methods work and to allow students to create their own methods of doing mathematics. She followed the assigned textbook for the course but tried to supplement the material by using manipulatives and asking questions "that allow the students to create their own ways of looking and solving certain problems." She was positive with her students but felt discouraged with the confusion they often experienced.

Rebecca's Mathematical Background and Perspectives

In college Rebecca completed two courses on mathematics for elementary school teachers and did not take any higher mathematics. Rebecca reported that what she liked about mathematics was that it is logical, factual, and challenging, and that there is "always a solid solution or answer." She qualified her answer by saying she liked "the logic of it once it gets into your head." She liked when things were logical and straightforward, but could not always understand them quickly. Rebecca found the least appealing thing about mathematics was associated with the way in which it was taught to her. She felt that she never knew exactly what she was doing because she could not keep up with all the procedures, algorithms, and theorems. She concluded by saying that mathematics "seemed too complicated to ever really be enjoyable."

The Professional Development's Mathematics Tasks and Background

My classroom video data are from the third semester of the five-semester professional development. The first two semesters consisted of a survey of geometry and algebra. By the third semester, the participants were well acquainted with each other and were accustomed to the routines of the professional development. The participants were familiar with the inquiry-based, collaborative problem solving in which they were expected to engage each week. The third semester's mathematics was trigonometry. Although my analysis focuses on participants' work on the *Placenticeras* task, I give a brief description of the previous tasks and activities to illustrate the background that Rebecca and her group members had as they began the *Placenticeras* task.

Each of the tasks completed during the third semester allowed the participants to build up important mathematical knowledge which shaped the way they approached the

Placenticeras task. The tasks used in this professional development were used to generate participants' mathematical ideas. The instructor did not solely lead the classroom activities; instead participants' inquiry drove the class and led to the creation and invention of mathematical ideas. The instructor's role was to then introduce formal mathematical notation and language generally accepted in the mathematical community.

Prior tasks led Rebecca and her group members to develop complex understandings of trigonometric concepts. Prior tasks led the participants to explore special triangles (30º-60º-90º and isosceles) and build up ideas about the relationships between the lengths of the sides of such triangles. Another task elicited the emergence, from participant collaboration, of a formula for finding arc length. The class was also introduced to the six trigonometric functions. Participants additionally used prior knowledge of inverse functions to discover how inverse trigonometric functions are used to find the measure of angles in a given triangle. The class was able to move from right triangle trigonometry to unit circle trigonometry and then work with graphing trigonometric functions. Especially applicable to this analysis was the fact that participants built ideas of the sine and cosine functions by looking at the behavior of the sine and cosine of angles as the central angle in the unit circle is increased (using reference triangles to make sense of the sine and cosine of the angles). Because of the way they built up their ideas about the sine function, some participants stated that the sine function was graphing the "motion of a circle," or in other words that $y = \sin x$ was related to the graph of a circle. The class also investigated patterns in exponential, linear, and quadratic functions. They investigated first and second differences in linear and quadratic functions and constant, or common, ratios in exponential functions. Thus, the

participants were able to identify several different kinds of functions and create equations to fit given data. As the class continued to build equations from given data, they were exposed to periodic data, for example, the daily number of daylight hours over two years.

On March 18, during the week immediately preceding the *Placenticeras* task, the participants were invited to consider a task regarding transformations of exponential, quadratic, square root, and power functions. Participants generated understandings of the general form of a quadratic function, $y = a(x-h)^2 + k$, in addition to building insights about translations of other functions. They were also presented a task that asked them to collect data from the motion of a swinging pendulum using CBRs[2] and then to model, with an equation, the pendulum's distance from a given point. Groups worked on the pendulum task for the remainder of the class time on March 18 and then on March 25 they discussed their ideas as a class to conclude the task. Through their collaborative group work and class discussion focused on the pendulum task, the participants developed a general form of the sine function, $y = a\sin(bx) + c$.

Rebecca's Specific Background in the Professional Development

Throughout the semester Rebecca built understandings of many of the mathematical, pedagogical, and epistemological concepts derived during her engagement in the professional development program. However, she sometimes felt inadequate while working on some of the tasks. In her write-up of a task in which she was asked to mathematically model a population, Rebecca wrote,

> I think I understand, but when it comes to pulling it all apart and trying to make an equation out of it, I'm lost. It takes me a <u>LONG</u> time.

---

[2] CBRs are small data collection devices made by Texas Instruments. CBRs connect directly to graphing calculators and can collect and record data such as distance, velocity, and acceleration.

Rebecca's lack of confidence in her own ability to work through difficult mathematics was very evident in her writing. She could understand the general idea of what was happening but found it challenging to articulate her ideas and come to a full solution.

However, Rebecca also had experiences that resulted in explicit reflections on her teaching practices. For example, one week the class discussed a proposed outline for planning mathematics lessons for their own classrooms. The outline included five phases: launch, exploration problem solving, shared mathematics, structured practice, and independent applications. In her notes that day, Rebecca commented on her practice and reflected on the implications that making changes might have on her students. She posed questions to herself such as "Do I really try to do this each day?" and "What if this were a week long process?" She resolved to "gear more toward student guided practices."

Rebecca was absent on March 18, the week immediately preceding the *Placenticeras* task, when participants collaborated on the function-transformation task and the pendulum task. She collected notes and data from her group, but struggled to work through these two tasks on her own. During the class discussion on March 25, she wrote her thoughts on the side of her paper.

> I'm thinking how terrible it is that I missed last week's class! All of these shared ideas would make so much more sense to me! I'm trying to do last week's task during all of this so that it clicks, but I'm too far behind. However, there are things that DO make sense, like how certain parts of the equation change what the line does.

Rebecca did not feel confident in her own understanding of modeling the pendulum's motion with a transformed sine function. When the *Placenticeras* task was introduced to the class, Rebecca felt significantly behind the rest of her group.

The *Placenticeras* Task

The *Placenticeras* task spanned two class sessions: March 25 and April 1. Rebecca, Brenda, and Christine were present for both class periods. Two group members, Lorri and Amber, played a part in the mathematical work on March 25 but were unable to attend class on April 1 because of parent-teacher conferences at their school. Another group member, Lyn, was unable to attend class on March 25 but was present on April 1. Although Lorri, Amber, and Lyn each played a part in the mathematical work of the group, their involvement is considerably less due to their absences. Thus the more-thoroughly analyzed participants are Rebecca, Brenda, and Christine.

For analysis, the five hours of participant work on the *Placenticeras* task have been broken up into meaningful, chronological sections. The narrative sections are as follows (See Figure 2).

Title	Video	Duration	Transcript Lines
Getting Started	25Mar04C	26:41	8-363
Developing Their Own Ideas	25Mar04C-25Mar04D	26:31 19:00	364-542 1-140
Getting Started Again	1Apr04A-1Apr04B	27:14 35:37	82-178 29-412
Brenda Moving the Group Forward	1Apr04B	8:31	452-526
Explaining Their Thinking	1Apr04C	13:39	1-175
Coming up with an Equation	1Apr04C	40:01	176-574
The Presentation	1Apr04D	9:10	1-85
Finishing Up	1Apr04D	17:13	136-336

Figure 2. The video of the focus group's mathematical work on the *Placenticeras* task, broken into meaningful, chronological sections.

In the following narrative data is presented directly from the transcripts. Such data is presented in this five-column format:

Line number	(Time)	Speaker	Speaker's direct utterances [with my annotations in square brackets]	Codes from my analysis

The fifth column is used to give the reader a sense of the coding, so its use is discontinued after the first few pieces of transcript.

Getting Started

With approximately ninety minutes remaining in the three-hour class period on March 25, participants were given the directions for the *Placenticeras* task (See Chapter 4 for a copy of the task). The class members were given the written task, blank transparencies, transparencies with photocopies of the shell on them, and markers. The instructor asked the class to use rulers, and not protractors, as they worked on the *Placenticeras* task.

Christine immediately suggested that the group standardize how they would orient the shell by agreeing on what would be considered the "top" of the paper. The group members then puzzled over how to proceed. They worked to agree on exactly where the center point should be and discussed various ideas of how to draw their rays. Christine suggested that they each draw their own randomly-placed ray and then copy all of the various rays onto one transparency to measure them.

Meanwhile, Rebecca had her own ideas. In the middle of the discussion on how to choose their rays, Rebecca voiced her interpretation of the task.

| 106 | (0:09:32.2) | Rebecca | Oh I think what they're going to try to do is show us the relationship between, 'cause see if we start here [pointing to the edge of the x-axis she had drawn on her spiral] and as our 'x' changes our 'y' [moving her hands around the spiral], uh, it's just going to show us pretty much how the. . | aha moment, predicting, relationship, making connections, figuring it out, creating, change, x, y, enthusiasm, collaborating, Rebecca speaking |

Brenda attempted to finish Rebecca's statement by saying "the unit circle." Rebecca

expressed frustration at not being able to verbalize what she was seeing. While the rest of

the group continued to discuss their rays, Rebecca tried again to articulate her thinking.

112	(0:10:01.3)	Rebecca	See because it'll be a circle [tracing her finger around the spiral] that's continually increasing.	circle, increasing, predicting, Rebecca speaking

Throughout the time she worked on the task, Rebecca often compared the spiral to a

circle with a consistently increasing or decreasing radius.

When Christine asked for Rebecca's ray a few minutes later, Rebecca mentioned

that she was no longer following what the group was doing. Instead, she said she was

"kind of taking off on my own."

The focus group continued their work of compiling each person's ray and

measuring. Accuracy was an important issue to these participants throughout the "Getting

Started" segment and throughout the rest of the task. Brenda noticed that Rebecca was

working on her own and initiated a conversation.

257	(0:20:34.7)	Brenda	What are you doing there?	collaborating, questioning
258	(0:20:37.8)	Rebecca	Well I'm just kind of. I was just wondering if we could create a wave that, never mind [putting her face in her hands]. You know how we did the circle and we did the rate of change. But the rate of change was always cons,,, not the rate of change, but the [closing her eyes and holding her fists in the air, extending her index fingers], how when the 'y' was higher the 'x' decreased [raising her left hand higher] .	collaborating, explaining, predicting, wave, circle, rate of change, comparing, x, y, decreasing, prior experience, reasoning, making connections, Rebecca speaking
259	(0:21:04.1)	Brenda	Yeah.	
260	(0:21:05.9)	Rebecca	And then when the 'y' began to decrease [slightly lowering her left	y, decreasing, Rebecca speaking

			hand].	
261	(0:21:09.0)	Brenda	Right	
262	(0:21:11.5)	Rebecca	The 'x' increased on the other side [moving her right hand sideways to the left]. So I'm just seeing what it does here [referring to the spiral], but I don't really know how to,	x, increasing, seeing, not knowing, figuring it out, Rebecca speaking
263	(0:21:20.8)	Brenda	show that?	showing
264	(0:21:21.9)	Rebecca	yeah.	

After only twenty minutes into the *Placenticeras* task, Rebecca mentioned creating a wave to model the spiral of the *Placenticeras* shell. The rest of the group worked on drawing rays and measuring the length of the segments from the center of the spiral to a point of intersection between the rays and the spiral. Christine then mentioned an important idea that later became significant to the group's solution.

289	(0:23:43.3)	Christine	I bet you if I could figure these [rays] the same distance [or angle apart] each time I measured it would be like a common ratio that it's [the length of the radii] decreasing by.	collaborating, decreasing, noticing patterns, predicting, ratio

At this time, Christine did not indicate why she suspected a common ratio in the radius measurements, but she did clarify that if she could use a protractor she believed she would find a common ratio between consecutive radii. Christine believed it would be possible to find patterns in consecutive radii if such radii were drawn to form congruent consecutive angles.

Christine remembered seeing a spiral task in the teacher's manual for the sixth grade mathematics text book and assumed it was the *Placenticeras* task. The group decided to look at the manual and see if they could find any hints on how to do the task. In the manual they found instructions for an enrichment activity that had a diagram of a

spiral circumscribed by a circle with twelve evenly spaced rays coming out of the center

of the spiral (Figure 3).

Figure 3. The elementary teacher's manual which the focus group studied.

Christine showed the diagram to the group and then began to read the instructions aloud,

"Draw a line perpendicular to radius one at point a. Draw a line perpendicular to radius

two at point b." At this point, the instructor of the professional development came over to

the group. Upon discovering what they were reading, she took the manual and asked the

participants what they had learned from it so far. They mentioned that the spiral was

circumscribed in a circle, but that Amber had already done that before seeing the book (as

had Rebecca). They also explained that the instructions said to draw perpendicular lines.

They determined that the purpose of drawing perpendicular lines was most likely to build

right triangles. As the instructor left, leaving the closed manual with the group, she

encouraged the group members to build their own ideas without using the manual.

Although it is difficult to determine exactly how the participants were influenced by

looking at the manual, some of their actions seemed connected to the diagram and the

instructions they had read. After this episode, some additional group members also traced

the spiral onto blank transparencies and circumscribed it with a circle. Drawing rays with

consistent spacing emerged later as an important strategy for working on the problem,

and although its importance may not have been immediately apparent to the participants, there were consistently-spaced rays on the diagram in the book. Christine seemed to be the one most influenced by the manual's instructions. This will be seen later in the description of how she decided to approach the problem after she read the manual.

Developing Their Own Ideas

For the next forty-five minutes the participants in the focus group split off and worked more independently. They continued to talk often to express their ideas or ask each other questions, but each individual began to pursue her own personal approach to the task. During the "Developing Their Own Ideas" period, individuals worked to build their own solutions with intermittent group or pairs discussions. The camera was able to capture some of their utterances and would occasionally zoom in on an individual's work. The video record is certainly incomplete, but none the less it is possible to piece together each individual's approach to the task.

When asked what she was doing, Lorri said she was taking "a different approach" instead of following the instructions from the teacher's manual. Using a dashed line, she traced the spiral onto a transparency. She drew x- and y-axes going through the center of the spiral, and drew randomly-placed rays coming from the center beginning with one or two rays per quadrant. Where her rays intersected the spiral, she dropped perpendicular segments to the x-axis (Figure 4).
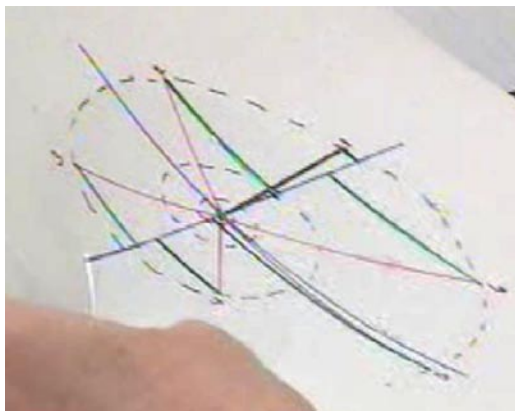
Figure 4. Lorri's approach to the *Placenticeras* task.

She then made a table of data with the rows labeled 1, 2, 3, … and the three columns

labeled "base," "height," and "radius(hyp)." She measured the sides of her right triangles

and recorded the lengths in the table. The base was the side of the triangle formed by the

x-axis, the height was the side of the triangle formed by the segment perpendicular to the

x-axis, and the radius, which Lorri also thought of as the hypotenuse, was the side of the

triangle formed by the ray drawn from the center, or initial point, of the spiral. Lorri filled

in the table by measuring the triangles in the counterclockwise direction, beginning in

quadrant one (Figure 5). Since Lorri was following the way the group had agreed to

orient the shell, the end of the spiral was in her third quadrant and then continued in the

counterclockwise direction as it spiraled inward. Thus, when Lorri began to enter data in

her table, she was not following the motion of the shell. Her first row of data began in the

first quadrant and she continued to follow the curve of the shell until she reached the ray

in the third quadrant that intersected near the end of the shell.

Figure 5. Lorri's table of data which she gathered by taking measurements of reference triangles around the shell.

After wondering about the information from the sixth-grade teacher's manual the group had briefly reviewed previously, Brenda decided to follow Lorri's idea. Using a dotted line, she traced the spiral onto a transparency and created x- and y-axes with the origin located at the center, or beginning, of the spiral. As Brenda drew rays coming from the center, she seemed to place them less randomly than Lorri; she had two rays per quadrant and they looked nearly evenly-spaced. She also dropped perpendicular segments to the x-axis from the points at which the rays intersected the spiral (Figure 6).

Figure 6. An early picture of Brenda's approach to the *Placenticeras* task.

Christine and Amber collaborated more extensively than other members of the focus group during the "Developing Their Own Ideas" segment of the video. Most of

their communication focused on accuracy. They were extremely concerned about the exact placement of the center, accurately drawing the circle around the spiral, and getting their two measurements for the radius of the circle to be the same.

Amber did not trace the spiral but used a transparency that had the shell photocopied directly on it. She drew a circle around the spiral and put x- and y-axes going through the center. She drew a few randomly-placed rays coming from the center, and where the rays intersected the outer portion of the spiral she dropped segments down to the x-axis, perpendicular to the x-axis (Figure 7).
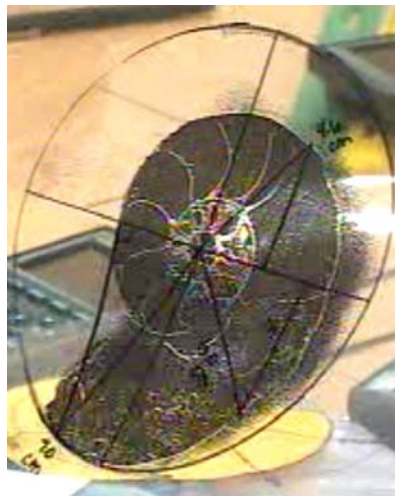
Figure 7. Amber's approach to the *Placenticeras* task.

Christine started over several times with new transparencies. She copied the spiral and circumscribed it with a circle. She placed x- and y-axes with origin located at the center of the spiral and drew carefully-spaced rays from the origin (Figure 8). Her drawings resembled the graphic in the elementary school teacher's manual. Christine often talked with other participants and did not complete much more on her transparencies during the "Developing Their Own Ideas" episode.

Figure 8. Christine's approach to the *Placenticeras* task.

Rebecca was often not in view of the camera during the "Developing Their Own Ideas" segment of the video. While she did many of the same procedures as the other group members, she did so quietly and spent most of her time appearing very thoughtful. She traced the spiral, circumscribed it with a circle, placed x- and y-axes with origin located at the center of her spiral, and drew segments randomly from the center to the edge of the spiral. She then measured the lengths of her segments and recorded them next to each segment (Figure 9).
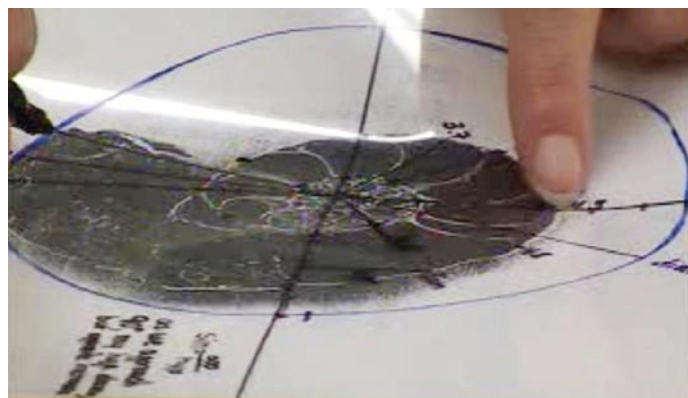
Figure 9. Rebecca's approach to the *Placenticeras* task.

While the rest of the group members were busy retracing their spirals and trying to perfect their measurements, Rebecca sat as if deep in thought. She stared at her work,

without talking often. At one point she placed her elbows on the table and rested her head on her fists, apparently puzzling over her transparencies.

Then, Rebecca drew what she predicted the solution to the task should look like (Figure 10).
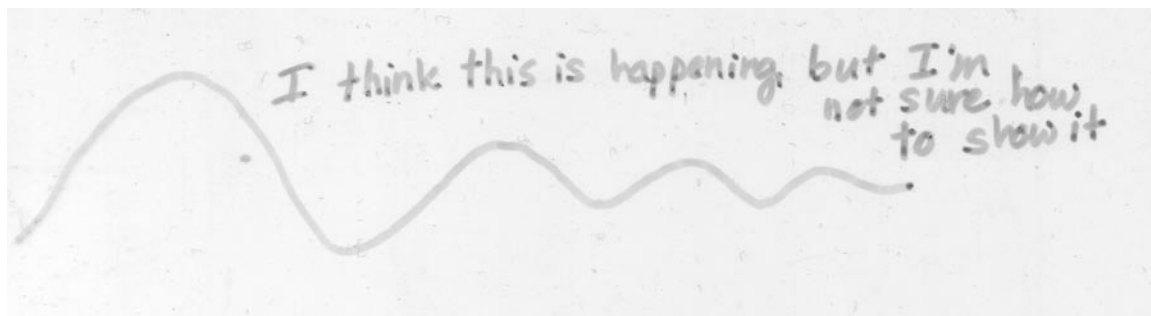
Figure 10. Rebecca's predicted solution to the *Placenticeras* task.

Brenda noticed Rebecca's drawing and began a discussion with Rebecca. Brenda and Rebecca looked through their notes to find their previous work with graphing $y = \sin x$ from the unit circle. Brenda said, "Sine is opposite over hypotenuse," and they continued to use that language. After describing the behavior of the regular sine function, Rebecca and Brenda tried to relate their ideas of opposite and hypotenuse to the case of the spiral.

529	(0:53:41.4)	Rebecca	Well this [spiral] is weird because my hypotenuse decreases as I approach ninety according to this. Hypotenuse increases, opposite decreases [actually, opposite increases, hypotenuse decreases].
530	(0:53:47.6)	Brenda	Right, so you have, you have. But your, your opposite [pointing to Rebecca's spiral and tracing the "opposite" side of one of the reference triangles] still is increasing, but it's increasing less. So it's [your opposite side] going to increase [tracing Rebecca's spiral from 0 degrees to 90 degrees], go down, increase [tracing from 90 degrees to 180 degrees], go down [continuing to trace the spiral], increase, go down. That's this [pointing to Rebecca's predicted graph of a damped sine wave]
531	(0:54:10.9	Rebecca	Right

| 532 | (0:54:11.7) | Brenda | I'm increasing, but I'm not increasing as much [tracing Rebecca's predicted wave as the amplitude decreases]. |
| 533 | (0:54:14.8) | Rebecca | Right. So what I want to know is how I show this [pointing to her predicted damped sine wave] in an equation. I know that's what it does, I just don't know how to show it. |

They did not address Rebecca's concern about describing her damped sine wave with an equation because Brenda decided to reorient her shell before working further. She retraced her spiral and drew a circle around it, but when she placed x- and y-axes through the center, she was careful that the very end of the spiral was touching the positive side of the x-axis (Figure 11).

Figure 11. Brenda's second approach to the task, in which she reoriented the shell so that the end of the shell touched the positive side of the x-axis.

A few minutes later, Brenda and Rebecca again discussed the behavior of the standard sine function. They referred to Rebecca's notes from a previous class session (Figure 12) to articulate exactly how the sine function increases and decreases, focusing on how the length of the opposite side of the reference triangle affected the shape of the curve (lines 92-94).
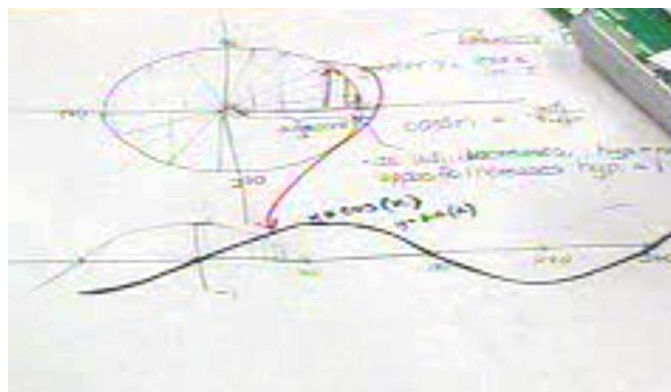
Figure 12. Rebecca's notes on the sine and cosine functions, from a previous class session.

92	(0:12:51.2)	Brenda	This is, this line is the cosine line [tracing the graph of $y = \cos x$ on Rebecca's notes], this line is the sine line [tracing the graph of $y = \sin x$ on Rebecca's notes]. Sine is opposite over hypotenuse, and the hypotenuse is one so it just kind of goes up. But what you're showing is opposite…[hypotenuse] is always one so it doesn't matter, but, in the circle. This opposite line [pointing to the opposite side of a reference triangle in the unit circle], side of the triangle, is increasing [tracing around the unit circle from 0 to 90 degrees] now it's decreasing [tracing around the unit circle from 90 degrees to 180 degrees],
93	(0:13:23.5)	Rebecca	Decreasing down to one-eighty [pointing with one finger on 180 degrees in the unit circle and with another finger at 180 degrees along the x-axis of the graph of $y = \sin x$]
94	(0:13:25.5)	Brenda	Now it's increasing but the negative direction [tracing from 180 to 270 degrees on the unit circle with left hand], now and then it's decreasing [pointing between 180 and 270 degrees on the x-axis of the graph of $y = \sin x$ with right hand], and then it's increasing the negative way. Then it's decreasing the negative way or increasing toward zero [tracing from 270 to 360 degrees on the unit circle with left hand and pointing between 270 and 360 degrees on the x-axis of the graph of $y = \sin x$ with right hand]. Then it will start all over again, as it goes around the circle. So now as this is going around in a circle [tracing a spiral with her finger], not only when this [the angle] starts to get bigger and [the opposite] starts to decrease, the opposite, but this [the hypotenuse] is also getting smaller, the hypotenuse, which is different than what we've had.

Brenda and Rebecca both noted that the distinction between the spiral and the circle was that the radius, or hypotenuse, in the spiral decreased while the radius of a circle remained constant (lines 94-95). They both believed that Rebecca's original prediction of a damped wave was correct (lines 97-98).

95	(0:14:09.5)	Rebecca	Well the hypotenuse [in a spiral] consistently gets smaller as we go [tracing a spiral shape on top of the unit circle in her notes].
96	(0:14:12.9)	Brenda	Yes and now as we pass ninety, this [the opposite side] is also going to decrease [tracing the unit circle from 90 to 180 degrees] and then increase in the negative direction [tracing from 180 to 270 degrees], but not at the same rate that it was before. And then it's going to increase, but at a smaller rate [tracing from 270 to 360 degrees].
97	(0:14:30.7)	Rebecca	So is this [pointing to her predicted wave graph] in fact what's happening? Ok cool
98	(0:14:35.5)	Brenda	From what I can see.

At this point, Christine and Lorri joined the conversation. Looking at Rebecca's damped-wave graph, Christine said, "So there should be an equation for that. Because remember Jim's that he put up there went small then big then small?" When the class worked on the pendulum task the previous class period, Jim, another participant in the professional development, had shared an equation that he had used to model the data from the swinging pendulum. He was able to create an equation whose periodic graph was a curve that increased in amplitude as it approached $x = 0$. Moments earlier Rebecca had asked her group if the *Placenticeras* task related to the pendulum task.

41	(0:04:16.0)	Rebecca	Isn't this [the *Placenticeras* task] sort of like the pendulum? Like out here [pointing to the end of the shell] is we're starting and then we're having a slower rate of change as we're going along [tracing the spiral from the outside in with her left hand while swinging her right hand like a pendulum with decreasing amplitude].

Even though Rebecca had missed the pendulum activity, she recognized that a damped

sine wave would be a good model for the data collected from the swinging pendulum.

However, the focus group had not tried to fit the entire set of data from the pendulum task

but had instead chosen to create an equation that would model the first cycle of the wave

only. Jim's equation was the only one presented in class that actually accounted for the

diminishing of the pendulum swings over time. Jim's equation was $y = \dfrac{-1.6}{x}\sin(2x^{1.6})$.

The participants tried to determine which parts of Jim's equation were responsible for the

specific behaviors of its graph.

Through their own approaches to the *Placenticeras* problem, Brenda and Rebecca

had each taken some "hypotenuse" measurements around the shell (Figures 13 and 14).
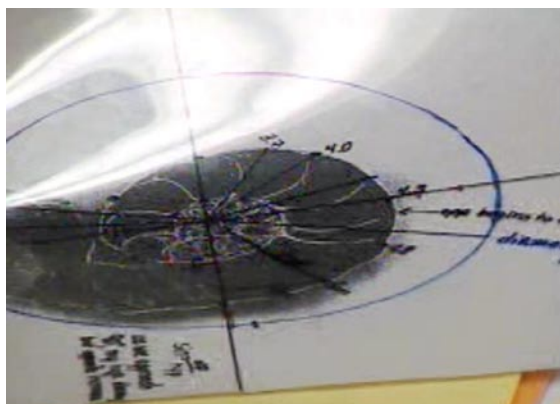
Figure 13. Rebecca's hypotenuse measurements around the shell.
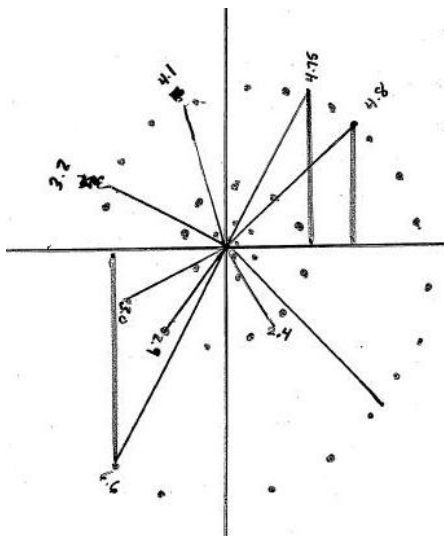
Figure 14. Brenda's hypotenuse measurements around the shell.

Rebecca thought she could see a common difference of .2 between consecutive hypotenuse measurements. Brenda and Christine both mistook Rebecca as talking about a common ratio of .2 between measurements. Christine thought that if there was a common ratio, then it must have something to do with the equation. The class session soon ended and the participants were all responsible for thinking about the task at home.

Getting Started Again

The next class started on April 1 with the focus group feeling confident they were on the right track with their solution so far. Christine and Brenda began by discussing and summarizing their ideas from the previous class period and their work at home throughout the week. Brenda described how she created her triangles by drawing rays every 30 degrees and dropping perpendicular segments to the x-axis from points where the rays intersected the spiral. She also described that she thought the graph should look like a wave that gets smaller. The two discussed Jim's equation and how it might be possible to adjust it to fit the wave they believed would be created by the spiral. They

continued to compare the spiral to a circle, trying to see how they could create something analogous to a sine wave for the spiral.

Brenda told the group that she had forgotten about the no-protractor rule during the week between classes (although it is possible that she might have disregarded the rule on purpose). So while working at home before the next class, Brenda had traced the spiral, circumscribed it with a circle, placed her x- and y-axes through the center, and then used a protractor to place rays coming from the center every 30 degrees. Early in the class period, Brenda decided to make a chart of data with columns "θ," "opp," and "hyp," so she could look for a "pattern in the ratios." Her "θ" column was filled in with angles from 0 degrees to 570 degrees at increments of 30 degrees. She began filling in measurements for the opposite and hypotenuse legs of the reference triangle starting on the row for 30 degrees. Brenda was able to fill in all the measurements for her "hyp" column by measuring the length of the segment from the center to the point of intersection of the ray with the spiral. Filling in the "opp" column proved more difficult for Brenda because she was not sure what was considered the opposite side for 90 degrees, 180 degrees, 270 degrees, and so on. Consequently, she left the table blank for the opposite side for each of those angles (Figure 15).

Figure 15. The beginning of Brenda's table of data which she gathered by measuring the opposite side and hypotenuse of reference triangles around the shell.

Christine agreed with Brenda's ideas about a sine wave and Rebecca's prediction from the previous class period; however, Christine continued to work on the task using an approach based on her reading of the teacher's manual. With a transparency on top of the photocopy of the shell, she had carefully circumscribed the outside edge of the shell with a circle, placed x- and y-axes through the center, and added rays that were approximately 22.5 degrees apart. Where the spiral intersected a ray, she would draw a segment perpendicular to that ray, from the point of intersection to the previous ray. In this manner, Christine created a series of right triangles (different from Brenda's collection of right triangles) whose legs came together to approximate the spiral (Figure 16). Christine then measured the lengths of the segments from the center to the point of intersection on the spiral and recorded them around the spiral.

Figure 16. Christine's use of right triangles in her personal approach to the *Placenticeras* task.

A few minutes later, Christine showed Lyn her work. Rebecca saw the different approach to creating right triangles and was very confused as to why Christine had used perpendicular lines in the way that she did. Christine explained that she needed to do it to help her visualize.

363 (0:32:18.9) Rebecca Why did you put perpendicular lines there [around your spiral]?
364 (0:32:21.6) Lyn You just make a right triangle
365 (0:32:23.8) Christine See, this is, yeah, I made a right triangle [holds up her

			transparency to Rebecca]
366	(0:32:26.4)	Rebecca	Just to find degrees?
367	(0:32:27.9)	Christine	I wanted to find what the ratio of decrease is in each little section [or triangle] like that.
368	(0:32:33.2)	Rebecca	Did you have to do that to find it [the ratio]?
370	(0:32:49.8)	Christine	[holds up her transparency to Rebecca] Rebecca, this is something that I remembered from one of the math units that one of the fifth grade did, it told, it said to do perpendicular lines, so that's why I was thinking that if I did that first, just to figure that [ratio] out, but now I'm going to do triangles [like the triangles in the unit circle] so that I can actually find out the degrees.
371	(0:33:07.9)	Rebecca	[holds up her transparency to Christine] Because I got the same thing as you did, simply measuring the distance from the center all the way around.
372	(0:33:16.1)	Christine	You're just one step ahead
375	(0:33:20.8)	Rebecca	What I'm saying is, I get scared when I see all of these perpendicular lines and I'm like [shocked facial expression], 'Why, why, why don't I have perpendicular lines? What's going on?'
376	(0:33:27.6)	Christine	No, it's how I had to vis, that's how I had to visualize it.

Brenda and Christine reviewed their two sets of hypotenuse data. Each had calculated the differences between consecutive measurements, but they noticed the differences were not constant nor did they show a consistent pattern. Christine figured that the accuracy of the measurements was the cause for most of the inconsistency. She also mentioned exponential growth, and that there would be a common ratio if the relationship for the hypotenuse data was exponential. Rebecca joined the conversation by asking about rate of change. Christine and Brenda discussed that since the differences between consecutive hypotenuse measurements were not constant, they wanted to investigate whether there was a common ratio between the data points. It is unclear at this point if the participants were referring to actually finding the ratio between two consecutive hypotenuse measurements or if they were thinking of taking the ratio of the consecutive differences from their calculations. Christine expressed that it might be

hopeless to find a common ratio because of the "messy" numbers, but Brenda argued that they could get the "general idea."

Rebecca was convinced that she had predicted the graph correctly; however, she stated that she still had no idea how to show the consistent decrease of the radius in an equation. She and Christine discussed her concerns. Christine mentioned that "this reminds me of an exponential equation because it's growing at a constant ratio." While a simple graph of the sine function would not be an adequate model, Christine shared that she thought using the sine somehow with an exponent would probably be the desired equation.

| 407 | (0:36:35.8) | Christine | But, as far as the equation goes, I think that it's a sine, it's an equation with sine but you've got to bring it to some type of exponent to show that you are growing at a constant rate, [makes an outward growing spiral in the air with her hand] that it's getting bigger. Does that make sense? So I don't know what the numbers would be, and I think that'll help us. That's why I'm trying to find the ratio right now. Now that's what I'm working on, but it's not working the way I wanted it to because of the measurements, but I think that if I could find the common ratio. . . Sine of x times, I bet, whatever our ratio equals, to, brought up to an exponent. |

Christine then suggested "playing around" with the calculator to investigate variations of Jim's equation, mentioned earlier, which contained both a sine and an exponent. Throughout the "Getting Started Again" segment and in all previous video segments, there had never been any concrete numbers to show that Rebecca's prediction was correct. The group members felt confident that they could "see" that her prediction modeled the data they were gathering, but they had a hard time thinking of how to create an equation.

Brenda Moving the Group Forward

Brenda completed her table of opposite and hypotenuse measurements (Figure 17). She was very excited to show the group "something cool." She began to graph the data from her "opp" column, calling it the "sine line" of the table.
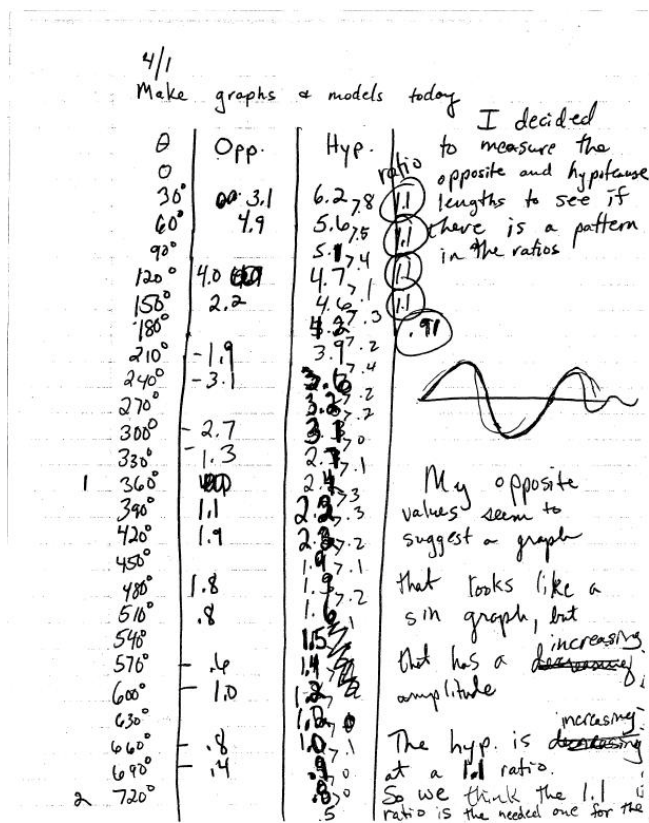
Figure 17. Brenda's completed table of data which she gathered by measuring the opposite side and hypotenuse of reference triangles around the shell.

Although she was missing data for every 90 degrees, she showed that the data graphed a wave that increased and decreased just like a sine wave, but that the measurements got smaller and smaller in magnitude. Everyone was very excited to see that Rebecca's prediction was definitely correct. Rebecca pointed to her predicted graph and said, "So this [predicted graph] is right! But the x-axis goes right through the middle."

Brenda suggested the idea that what made the graph of her hypotenuse data different from a sine curve was that the amplitude was decreasing.

461   (0:44:00.1)   Brenda   So my amplitude is always coming down, my amplitude is decreasing

The group members remembered that during the pendulum task, another group had come up with a general equation for sine curves, $y = a\sin(bx) + c$, where '*a*' was the amplitude. Christine drew their attention to the general equation and discussed her ideas for what should go in place of the '*a*.'

487   (0:46:08.4)   Christine   Because [points to her notes] the amplitude is, in this [general sine equation], is '*a*,' so it's the one in front of sine. So I wonder if we put like a number in there, like x or something, but then to the negative two or something like that.

The group members were working toward the idea that if they could figure out an equation for the decrease of the amplitude, then they could multiply that by the sine of x to get the equation of their damped sine curve.

Brenda's contributions in the "Brenda Moving the Group Forward" segment were crucial to the mathematical work of the group. She was able to finally demonstrate that Rebecca's prediction did model the data. Also, she helped the group progress toward creating an equation because she brought forward the idea of amplitude.

The participants agreed that if the hypotenuse was growing exponentially, then they would need to find the common ratio to use in their equation. They decided to divide the hypotenuse measurements from Brenda's table to find their ratio. Brenda was eager to show the group that dividing consecutive hypotenuse measurements revealed a common ratio of 1.1. Brenda believed that she had found the ratio that represented moving from

the outside of the shell in, although she had actually divided in the opposite order and had found the ratio for beginning in the center of the spiral and moving out.

Explaining Their Reasoning

As the group members were enthusiastically discussing their breakthroughs, the instructor and a participant from another group came to see what they were doing. Brenda, Rebecca, and Christine were able to explain their work to this point. Brenda explained that she measured her angles every 30 degrees so that she would have a few points in each quadrant without gathering too much data. She described how she measured her radii and noticed a decrease. She also noticed that a spiral is similar to a circle and so she formed triangles and measured the opposite sides to connect to her prior experience with sine curves and the unit circle. Rebecca and Christine then explained that the spiral is a circle with the hypotenuse consistently decreasing. They discussed how the sine curve comes from the y-values on the unit circle, and on the spiral those y-values had a narrower change as they moved inward. They expressed how that meant that the amplitude of the wave for the spiral was consistently decreasing and so they needed to put something in place of the '*a*' on the equation $y = a\sin(bx) + c$. They then described how they had found a common ratio in the hypotenuse measurements, which means that the equation is somehow exponential. Speaking of their collaborative efforts, Rebecca said, "Well, we kind of did it on our own for a long time, and then for the last like fifteen minutes we kind of all came together and went 'Huh? We all have similar ideas here.' But we just, we're all stuck on how to show that increase." Then pointing to her predicted graph she said, "This is what I envisioned happening in the beginning." The participants

then explained that they did not have an equation yet, but they were thinking about using

a sine function, a ratio, an exponent, and possibly using a version of Jim's equation.

Coming Up with an Equation

After all the discussion about needing to somehow use ratios, exponents, and sine

to create an equation, Rebecca said "Let's try it out; let's see what happens when we do."

The group members were still unsure about where to put the ratio, exponent, and sine

function, and what should go in the exponent or inside the argument of the sine function.

Brenda also wondered if they should move from the outside to the inside of the spiral,

graphing a damped sine curve with decreasing amplitude, or if it would be better to move

from the inside of the spiral to the outside, thus graphing a damped sine curve with

increasing amplitude. They determined that it would be better to work from the inside of

the spiral to the outside because that could predict growth if the shell had been able to

keep growing. Brenda found the common ratio of .91 by dividing consecutive hypotenuse

measurements in the opposite order of what she had done previously. The group

concluded that .91 was the common ratio when working from the inside of the spiral to

the outside, or the one used to predict growth.

The group members began putting equations into their calculators to see what the

graphs would look like. Christine entered $y = .91^x \sin x$ into her calculator and was

excited when it graphed a damped sine curve; however, the graph had decreasing, and not

increasing, amplitude. They thought that introducing a negative might make the graph

behave as they wanted, so Rebecca tried $y = -.91^x \sin x$ but it still graphed a damped

sine curve with decreasing amplitude. Christine then tried $y = .91^{-x} \sin x$. All of the group

members were extremely excited to see the damped sine curve with increasing amplitude that resulted.

At this point, groups began presenting their various approaches to the problem and their solutions or works-in-progress. The focus group was so energized about their breakthrough that they rarely paid attention, but instead each person tried to get their own calculators to graph the equation in an appropriate window.

Another class member had brought a large shell to class that was similar to the one in the photo. The focus group decided to work with that shell to see if it would have the same equation. Brenda traced the shell, placed x- and y-axes through the center, and drew rays at approximately every 30 degrees. She then measured the first two hypotenuse measurements for the new shell. The entire group was ecstatic when the ratio between the two measurements was .91.

Throughout other groups' presentations, members of the focus group would quietly talk to each other about issues brought forward in the presentations. Brenda and Christine talked about placing rays a consistent angle apart to get accurate data. They were also confused when another group, who had placed their rays every 15 degrees, got a different ratio from their data. When one group used a simple exponential equation to describe their data, Brenda expressed that she did not understand how an exponential function, without a sine component, could describe a spiral. This was a recurring idea, that modeling the circular motion of the spiral required using a sine function because the sine function was somehow connected to the motion of a circle.

The Presentation

Once the participants were able to graph a damped sine curve with increasing amplitude, they were ready to present. Brenda and Christine dominated the presentation with Rebecca only chiming in a few times; however, they brought together everyone's ideas very well and credited many ideas to others or to the group as a whole. Essentially, all the prior work done on the task was brought together to form one solution for the group.

In their nine minute presentation, Brenda began by explaining how they started with tracing the spiral, placing rays at 30 degree increments, and measuring the hypotenuse and the opposite side of the reference triangles. Rebecca showed her predicted graph and described it as a sine with consistently decreasing amplitude. Showing Brenda's table of data, Christine explained that the data fit their prediction. Because they found a common ratio in their hypotenuse measurements, they knew that the data had an exponential relationship with a ratio of 1.1. Brenda then explained that they reversed their thinking to start at the center and go out because they wanted to be able to predict growth. This reversal of thinking led to the new ratio of .91. Christine reminded the class that exponential functions are a ratio raised to the x. Then Brenda explained that they wanted to use the sine as well so that they could get the circular motion. She reminded the class of the general sine function $y = a\sin(bx) + c$ that was previously developed by another group. Since the 'a' represents the amplitude of the graph, they would need to put something in place of 'a' to account for the consistently decreasing amplitude. Rebecca put the expression $.91^{-x}\sin(x)$ on the board for the class (Figure 18).

$$.91^{-x}\sin(x)$$

Figure 18. The equation the focus group wrote on the board as their solution to the *Placenticeras* task.

Christine explained that they needed the –*x* in the exponent to get the graph to go "from small to big." Brenda then described their work with tracing the other shell and finding the ratio was again .91. Brenda explained that they believed that the equation would be true of all shells. Rebecca, Brenda, and Christine were all extremely enthusiastic about their solution to the task and were very eager to share.

After the group finished presenting, the instructor decided to discuss the $.91^{-x}$ with the class. She pointed out that from what they already knew about exponents, they could rewrite $.91^{-x}$ as $(\frac{1}{.91})^{x}$. The focus group was pleased to see that $\frac{1}{.91}$ was the same thing as 1.1, so $.91^{-x} = 1.1^{x}$. The instructor helped the class understand the different kind of ratios for exponential growth and exponential decay. Christine was validated to hear about the difference between exponential growth and exponential decay because she had been slightly confused all along about how to describe an exponential relationship where the data was getting smaller instead of growing larger. Brenda admitted that she had gotten things flipped around and apologized, but the group was happy that they had had the ratios all along, even if they had been mixed up. To this, Rebecca replied, "Are you kidding me Brenda? You were like the missing link for all of our ideas."

Finishing Up

During the last group's presentation, Christine realized that the instructions in the task mention that the actual shell is half the size of the photocopy. After some deliberation about whether they should multiply or divide by two, the focus group agreed that they should divide their equation by two to model the actual shell. Feeling that they had successfully completed the task, the participants expressed enthusiasm and satisfaction with their work. Rebecca felt especially confident in her work on the task. She told her group members,

336	(0:37:40.4)	Rebecca	You know what though, this is the first time that I have a beginning and an end to my write-up, in my head. You know what I mean? I usually have bits and pieces and I'm like, 'And this is where I faded. And this is where I came back in.' Really, because I, because so seldom do all the connections come together. But today was cool because we all had different elements [holding her hands 6 inches apart] and we all kind of went 'Now what if we did this?' [bringing her two hands together] And we all experimented [drumming her two hands together on the table] and pa-pum. That was cool.

Rebecca's Interviews

Rebecca's interviews were an important part of my analysis of Rebecca's transformative experiences in the professional development. I developed a set of Interview codes based on Rebecca's responses to the interview questions and used these codes to tie Rebecca's self reports to the actual mathematical work she and her group performed on the *Placenticeras* task. I give a brief description of Rebecca's two interviews and her exit survey before describing the codes that emerged.

Rebecca's First Interview

Rebecca's first interview took place after the fourth semester of the professional development. Because the mathematics content of this semester was calculus, a few of Rebecca's responses centered around calculus. Rebecca admitted that she had never known what calculus or trigonometry were before, but enjoyed seeing the connections between the two subjects as they learned about them in the third and fourth semesters. She related a "revelation" she had in her own elementary classroom when she saw how finding volume related to calculus concepts from the professional development. She reported having frequent "aha moments" throughout the course of the professional development. When asked about any calculus concepts that really made sense or "stuck out" to her, Rebecca described how she liked using "tiny rectangles" to find the area under a curve because it was amazing how such "elementary geometric principles" could be used in mathematics that had, at one point, seemed impossible to her.

Rebecca explained that she now thought about mathematics "completely differently" after her years of participating in the professional development. She enjoyed greater freedom in mathematics as well as being able to look at mathematical ideas from different angles, or multiple viewpoints. Although she described herself as "one of the slowest thinkers" in the professional development, she said that her confidence in her own abilities to reason and understand mathematics had grown tremendously. She felt her increased confidence had changed the way she interacted with mathematics.

Rebecca asserted that she had changed her ideas about how she learns mathematics and about how children learn mathematics. Changing her perspectives on learning led her to change her teaching practices. She described the process she had

experienced as she changed her teaching practices. The professional development program began halfway through Rebecca's first year of teaching. Rebecca explained that she "went by the book" and "did things step by step" in the beginning of her first year (before the professional development). In the beginning of the professional development (near the end of her first year of teaching), Rebecca felt excited about new ideas for mathematics instruction, but did not feel confident enough to try them regularly. She described doing "little snippets," holding her breath, and hoping for success. While she did attempt some learner-centered practices near the end of her first year, she did not make them regular practices in her classroom. In her second year of teaching, Rebecca reported that she "went. . . nuts" and "threw away the book." She tried many new things, and met many successes and failures. At times during the second year, Rebecca would experience a lack of confidence in her mathematical understanding of certain concepts. At these times, she admitted going back to the "easy way" of "step by step" teaching. Rebecca reported the most success was coming from her third year (of which she was only halfway through). She felt much more confidence in her mathematical ability. She said that her students believed she knew everything about mathematics and many of them described mathematics as their favorite subject. She described with enthusiasm that her experiences in teaching mathematics were getting better each year, especially since she was finally "creating lessons the way I know I want to teach."

Rebecca's Second Interview

Rebecca's second interview took place at the end of the fifth and final semester of the professional development. Rebecca began with the assertion, "My classroom practices are drastically changed from this program." She expressed much excitement

about being able to describe the specific changes she had made. Rebecca described her teaching as being exactly like the teaching that was done by the instructor in the professional development program. She would "throw problems out" and let the students think. She also would allow her class to veer off into different mathematical topics than the ones she had planned, even if she did not know all the answers. Instead of curbing this further exploration as she had done during her previous year of teaching, Rebecca allowed her class to "take off" because she knew that they could all look at the mathematics from multiple viewpoints and they could figure it out together. Rebecca reported that she had created her own tasks "like crazy." Rebecca recounted many successes that resulted from her changed practices. She described students having increased confidence academically, more students loving mathematics who had hated it in the past, parents who were supportive and impressed, and test scores that were "looking really good." Rebecca seemed especially pleased with her students' increased confidence and ability to look at mathematics and understand mathematical concepts, even above the grade-level curriculum.

Rebecca offered suggestions for helping other teachers change. She felt that the first thing teachers needed was to be motivated to change by seeing the possibilities that come from improved practices. She felt that if teachers could just "see what the kids can do" and "how [students] can create their own way to [a] problem," then they would get excited about giving students the freedom to reason mathematically. She suggested letting teachers start with one lesson, analyze the various approaches, and see what happens. She felt that if the vision could get started, there would be hope for substantial changes in practices.

When asked what her favorite task was from the professional development, Rebecca responded with the *Placenticeras* task.

| 16 | (0:09:29.7) | Rebecca | I loved the placenteras [sic] task because that was something that I had no scaffolding, I had no previous knowledge and it was just kind of like thrown in the deep end. But it was so cool because that was probably the first task that really came alive for me that made me think like wow I do not have to necessarily be a college level mathematician to understand how math works. |

She reported enjoying it because she had "no scaffolding" or "previous knowledge" and was asked to jump in and figure out the task. The *Placenticeras* task was the first task that really "came alive" for her and made her think that she could "understand how math works," even though she was not a "college level mathematician." She enjoyed how each member of her collaborative group tried to do the task on their own, each of them failed, but then they kept going and put their ideas together, and it all worked together. She had so much fun with the *Placenticeras* task that she worked on a sine wave with her own students.

Rebecca said that she really just wanted her students to see how mathematics works because she had never seen it as a student. Rebecca had expected to simply learn more mathematics from the professional development, but instead she was pleasantly surprised to experience a paradigm shift.

| 20 | (0:12:52.5) | Rebecca | I thought I was just going to learn more math, not a whole way of teaching and thinking about it personally, and so I was like 'wow.' |

It had been a difficult experience, but she was now able to see that mathematics "all comes together."

Rebecca's Exit Survey

At the end of the professional development, all of the participants were required to respond to an exit survey that was nearly identical to the initial survey. The exit survey contained all of the same questions as the initial survey, with five new questions at the end. (See Appendix D for Rebecca's entire exit survey.)

When asked three necessary qualities of an excellent mathematics teacher, Rebecca replied, "patience, open mindedness (considering multiple ways of looking at a problem), and personal prep. and study (KNOWLEDGE)." Of these three qualities, Rebecca felt strongest in patience and weakest in her own mathematical knowledge. She wrote, "I feel I have so many abilities to teach math well, but I've never had a true 'math mind.' I am starting to improve this, however."

Rebecca asserted that she had made many successful changes is her mathematics teaching. She wrote, "Math is my favorite subject to teach which was originally my most dreaded subject." She described her teaching practices as "a confidence builder approach that reminds each student that they are completely capable to learn math." She focused on giving the students the "freedom and time to think through problems and discover." Rebecca also maintained that her students connected mathematics to other subjects, problem solved constantly, and felt all right getting a "wrong answer" as long as it kept directing them toward a right one. She was proud of her students for discovering so many things and even figuring out mathematics "beyond their leveled curriculum."

As in the initial survey, Rebecca claimed that the way she was taught mathematics as a younger student was the least appealing thing about mathematics. She further elaborated that, as a student, mathematics had been presented as "segmented, difficult to

understand, technical theory. . . It was approached as a secret language that only 'smart'

people could speak." Rebecca described that the professional development program made

her realize that is not the case with mathematics, but instead, "It all works together! . . .

It's actually a web of patterns." Rebecca felt that she still did not know "enough"

mathematics, and stated in order to change she would "Learn more! Think more! Practice

more!"

Rebecca described her view of her upcoming role as a numeracy coordinator at

her own school. She felt that she would be "a motivator and an example." She wanted to

show other teachers what successes came from her new teaching practices. She felt she

could "help prepare tasks, open up new ideas, and motivate participation." Rebecca felt

confident that she could help other teachers change without too many challenges because

"once people start to see results, they'll come to appreciate these approaches and use

them."

<div align="center">Analysis</div>

With a sense of Rebecca's background from her initial survey, a basic

understanding of the mathematical work done by the focus group on the *Placenticeras*

task, an overview of Rebecca's responses in her interviews, and complete coding of all

the data, the next step was to sort the codes and seek to recognize patterns. As previously

mentioned, in Chapter 4, this sorting led to the development of core theoretical categories

(Charmaz, 2006) which were important in analyzing Rebecca's personal transformations

and transformative experiences throughout the professional development.

Core Theoretical Categories

   I wanted to identify what had been transformative for Rebecca in the professional development program, specifically what had led her to change her perspectives on mathematics, learning, and teaching. So four core categories were developed: *Transformations*, *Mathematics*, *Learning*, and *Teaching*. The category *Transformations* clustered all the codes that were associated with the specific changes Rebecca reported having made or that had been observed throughout the professional development program. The category *Mathematics* incorporated all codes related to Rebecca's perspectives on mathematics. *Learning* was the category for tracking codes associated with Rebecca's epistemological perspectives on how she or others learn mathematics. The category *Teaching* encompassed codes relating to what Rebecca considered effective mathematics teaching as well as her reports of her actual classroom practices.

   To keep the work grounded in Rebecca's utterances, Rebecca's interviews were used to frame the analysis. I began sorting the four core categories based on the coding from the interviews. Using the search function in Transana, I formed a collection of all clips from Rebecca's interviews that corresponded with the codes for each of the four categories. For example, to build the category *Transformations,* all clips with the code "transformation" were put into a collection. For the categories *Mathematics*, *Learning*, and *Teaching*, collections were created using the codes "mathematics," "learning," and "teaching practices," respectively. Once the collections were complete, I was able to search for all the codes from the clips in these core category collections, along with their frequency, thereby sorting the Interview codes into the four core categories. Initial analysis showed that within each category, the analytically powerful codes were those

that occurred with the highest frequencies. The analysis of the four core categories was

then focused on the collections of high frequency codes (Figure 19). It is important to

note that although the following frequency tables helped illuminate important

relationships, it was qualitative analysis of the data that led to the eventual findings,

which are discussed in Chapter 6. (In the following figures, "*n*" represents the number of

clips within Transana and "*f*" stands for the frequency of each code.)

Transformations		Mathematics		Learning		Teaching	
n=48		n=27		n=22		n=27	
Code	*f*	*Code*	*f*	*Code*	*f*	*Code*	*f*
Teaching practices	20	Learning	15	Mathematics	15	Transformation	20
Confidence	14	Transformation	14	Transformation	12	Learning	11
Mathematics	14	Enthusiasm	11	Teaching practices	11	Students	10
Reporting on success	14	Teaching practices	9	Confidence	8	Mathematics	9
Students	14	Personally	9	Enthusiasm	8	Reporting on success	9
Enjoying	13	Confidence	8	Students	8	Confidence	7
Learning	12	Ownership	8	Understanding	8	Enthusiasm	7
Personally	12	Multiple viewpoints	6	Ownership	6	Telling	6
Enthusiasm	8	Making connections	6	Reporting on success	6	Allowing	5
Prior experience	8	Prior experience	6	Ability	5	Authority	5
Ownership	7	Understanding	6	Making connections	5	Persisting	5
Authority	6	Enjoying	5	Personally	5	Personally	5

Figure 19. The four core theoretical categories, *Transformation*, *Mathematics*, *Learning*, and *Teaching*, which were developed through searching Rebecca's interview video clips for the occurrence of Interview codes in conjunction with the core category codes "transformation," "mathematics," "learning," and "teaching practices," respectively.

In the previous figure, recall that each of the four categories represents a separate

search within Transana. For example, the *Transformations* category was constructed by

grouping all of the interview video clips that were coded "transformation" and searching

for the frequency of all the additional codes corresponding to those video clips. Thus a

particular clip in which Rebecca described, for example, a transformation in her

perspectives on mathematics would certainly be sorted through twice, once within the 48

clips in the *Transformations* category and again within the 27 clips in the *Mathematics* category. Hence, many video clips were used in multiple categories, but having the four separate categories was useful in teasing out important relationships within the data.

An additional search was performed to locate individual clips from Rebecca's interviews that were coded with all four codes ("transformation," "mathematics," "learning," and "teaching practices"). I believed that examining clips that simultaneously corresponded to all four core categories could provide insight into any experiences that had been particularly transformative. The search yielded five clips which are discussed in Chapter 6.

Further Sorting and Analysis

While Rebecca's transformations of her perspectives on mathematics, learning, and teaching were of specific interest, I realized that her responses in her interviews might have brought forward other important points relating to her transformative experiences in her professional development. I searched all the interview clips from both interviews and found the most frequent codes. This search yielded the information in Figure 20.

Both interviews	
n=107	
Code	*f*
transformation	48
confidence	30
mathematics	27
teaching practices	27
enjoying	24
students	24
enthusiasm	23
learning	22
reporting on success	21
making connections	17
noting inadequacy	16

personally	15
prior experience	15
understanding	15
ownership	11
ability	10

Figure 20. The most frequent Interview codes from the interview video clips.

These frequency searches provided interesting results, such as confidence as a significant issue in Rebecca's experiences in the professional development and in her own transformations, as well as the juxtaposition of "reporting on success" and "noting inadequacy." These results are discussed in detail in Chapter 6.

Rebecca reported in her second interview that the *Placenticeras* task had been, for her, the most compelling of all the tasks from the entire professional development program. She also reported that her classroom practices were "drastically changed" and that she had developed an entirely new way of thinking about mathematics, both as direct results of the professional development. Because she reported the task as being so significant to her experience in the professional development program that led to such transformations in her perspectives on mathematics and teaching, I decided to search the *Placenticeras* task data for any other patterns in the codes that could explain why the experience had been so influential. To make the analysis most effective, the codes were separated into three categories: *Placenticeras* task codes focused on behaviors, *Placenticeras* task codes focused on mathematical terms, and Interview codes. While the *Placenticeras* task codes focused on mathematical terms were helpful in understanding the mathematical work of the group on the *Placenticeras* task, they proved to not facilitate any further analysis of the transformative nature of the task for Rebecca. Thus, the results of that search have been omitted. A search for the frequency of codes yielded the following results (Figures 21 and 22).

Placenticeras task codes: Behaviors	
n=594	
Codes	f
Rebecca speaking	331
collaborating	304
questioning	158
explaining	73
describing	60
comparing	57
showing	49
not knowing	46
asserting	44
predicting	31
measurement/measuring	30
seeing	27
accuracy	22
playing around with it	19
noticing patterns	18
plugging in	18
process	17
working on my own	17
verbalizing	7
missing link	5
simple	2

Figure 21. The frequency of *Placenticeras* task behavior codes in the *Placenticeras* task video clips.

"Rebecca speaking" occurred in 56% of the *Placenticeras* clips, but it was not because Rebecca truly spoke half the time during the problem solving. The particularly high frequency of the code "Rebecca speaking" is due, in part, to the fact that I coded every utterance of Rebecca's (including off-task and irrelevant comments) with this code in order to track exactly when Rebecca did or did not verbalize. Utterances from other participants were only coded if they were relevant to the mathematical work of the task. Although I found that it was not necessarily beneficial to have coded Rebecca's irrelevant comments, I have decided to keep the clips and codes as a part of my data for the sake of practicality.

Interview Codes
n=594

Code	f
prior experiences	33
enthusiasm	20
excited	15
figuring it out	15
making connections	13
enjoying	11
confidence	10
understanding	9
noting inadequacy	8
reasoning	7
reporting on success	7
aha moment	7
frustrating	7

Figure 22. The frequency of Interview codes in the *Placenticeras* task video clips.

The search of the Interview codes in the *Placenticeras* task video clips showed that "prior experience" was the most frequent code. Especially surprising was how often the codes "enthusiasm," "excited," and "enjoying" were coded in the data. All the results of sorting and searching my codes are discussed in Chapter 6.

CHAPTER 6: FINDINGS

Core Theoretical Categories

Data show that Rebecca experienced transformations in her perspectives on mathematics and learning, in her teaching practices, and in her confidence as a mathematics learner. The four core theoretical categories, *Transformations, Mathematics, Learning, and Teaching,* provide frameworks for understanding Rebecca's transformations.

Rebecca's Transformed Perspectives on Mathematics

From the beginning to the end of the professional development, Rebecca made significant transformations in the way she viewed mathematics. The details of Rebecca's transformed perspectives on mathematics are described here.

The relationship between the *Mathematics* and *Transformation* categories showed that Rebecca experienced a transformation in how she personally viewed mathematics. Rebecca reported in her second interview that she did not just learn mathematics, but learned a lot *about* mathematics from the professional development. Recall that she said "I thought I was just going to learn more math, not a whole way of . . . thinking about it personally" (line 20).

The specific details of Rebecca's transformed perspectives on mathematics were made evident in analyzing the *Mathematics* category. Frequent codes in the *Mathematics* category were "multiple viewpoints," "making connections," "prior experience," and "understanding" (each occurring in 22% of the *Mathematics* clips). In her exit survey, Rebecca described that what she liked most about mathematics was "how it all works together. . . It's actually a web of patterns." Rebecca came to see that mathematics ideas,

procedures, and concepts all "work together" in an interconnected way. She also reported by the end of the program that she thought that with mathematics "there is so much to be discovered!" Rebecca's perspectives on mathematics at the end of the program were in stark contrast to her initial perspectives on mathematics that focused on procedures, algorithms, and theorems, when Rebecca felt that mathematics was "too complicated to ever really be enjoyable."

Rebecca's new conceptions of mathematics were centered on building webs of personal understanding where all the pieces work together, as meaningful connections are made to prior experience. She felt that there were multiple ways to personally approach problems and think about mathematical concepts, and she also experienced that "multiple viewpoints" can be found by collaborating, whether in teacher-and-student interactions or student-and-student interactions. Rebecca discussed both of these interpretations of "multiple viewpoints" in her interviews. In the first interview she said, "I've just thought about mathematics completely differently. I look at it from so many different angles, and I really like that freedom" (line 78). In the second interview Rebecca described how she was able to move beyond being afraid to let students explore mathematics that she did not fully understand. She said, "Who cares that I don't know, if we [my students and I] look at it from different angles we all find out" (line 8). Rebecca was able to view mathematics from "many different angles" herself, and she realized that collaboration brought additional opportunities for approaching concepts from multiple viewpoints to reach unified solutions. Through her experiences in the professional development, Rebecca came to view mathematics as an interconnected web of patterns with multiple ways of viewing and approaching concepts; she also believed mathematical activity

included making meaningful connections, bringing together multiple viewpoints through collaboration, and building on prior understanding. These views are consistent with the kinds of activities in which she engaged during the professional development program.

Rebecca's language in describing her transformed perspectives revealed an important connection between personal agency (Walter & Gerson, 2007) and Rebecca's views on mathematical activity. Rebecca described the newfound "freedom" she enjoyed as she was able to look at mathematics from "different angles." This freedom implies an exercise of personal agency on the part of mathematics learners. In the professional development program Rebecca and the other participants were afforded opportunities to exercise personal agency in choosing to view mathematics from multiple viewpoints and to personally engage in problem solving using their own approaches. A theme of personal agency emerged from the data and describes well the conditions of the mathematics learning of the professional development, Rebecca's transformed perspectives on mathematical activity, and Rebecca's transformed teaching practices.

"Enthusiasm" (41%), "personally" (33%), and "ownership" (30%) were also frequent codes within the *Mathematics* category. Rebecca certainly conveyed a lot of enthusiasm when expressing her new perspectives on mathematics, and changing her perspectives had consequences that were personally meaningful to her. As Rebecca came to view mathematical activity as something accessible that she could engage in, she began to take ownership for her mathematical understanding. She was able to make her own connections and use her own approaches with mathematical tasks, whereas before the professional development, she thought correct mathematical thinking was generally told to students by an authoritative teacher. Rebecca's mathematical ownership and the

use of her own personal approaches to problem solving reiterate the personal agency

Rebecca enacted as she engaged in learning mathematics in the professional

development.

Rebecca's Transformed Perspectives on Learning

    The professional development setting provided valuable experiences for Rebecca

to transform her perspectives on mathematics learning. Although closely connected to

Rebecca's views on mathematics, the transformation of her views on learning was an

additional transformation that is discussed in this section.

    The close relationship between the *Learning* and *Transformations* categories

revealed that Rebecca transformed her perspectives on learning. Careful analysis showed

that Rebecca's transformation in her perspectives on mathematics learning was strongly

connected to her transformation in her perspectives of mathematics. As was discussed

previously, Rebecca changed to view mathematical activity as making meaningful

connections, bringing together multiple viewpoints through collaboration, and building

on prior understanding by exercising personal agency. Thus, her perspectives on learning

changed to consider learning mathematics as engaging in such mathematical activity.

    "Students" (36%) was a frequent code in the *Learning* category. Rebecca was a

conscientious teacher who was concerned about her students' learning. Rebecca's

concern for her students' learning was consistent with the view that teachers are

professionals who strive to provide the best learning opportunities for their students

(Loucks-Horsley et al., 1998). Although "personally" (23%) was also a frequent code,

Rebecca discussed her ideas about learning more frequently in regard to her students than

about herself.

"Understanding" (36%), "ownership" (27%), "ability" (23%), and "making connections" (23%) were also frequent codes in the *Learning* category.   In her first interview Rebecca discussed her mathematical ability and what she had come to realize.

82   (0:36:20.9)   Rebecca   And then I just realize hey, no I pretty much had this ability [to engage in mathematics] all along, just wasn't really invited to find it out I guess. [chuckles]

As Rebecca's perspectives on mathematics changed to include making meaningful connections and developing personal understanding of mathematics, she realized that she did have the ability to learn mathematics and engage in important mathematical activity. Prior to the professional development Rebecca had felt unable to learn mathematics. She came to realize, through her experiences in the professional development program, that the best invitations for mathematics learning had not previously been extended to her. The professional development setting invited Rebecca to exercise her agency in approaching mathematics tasks, thus developing ownership for her understanding and ability. As she accepted the invitation to engage in mathematics, she found that she truly was able to do so.

Rebecca's Transformed Teaching Practices

In this section I provide details of Rebecca's transformed teaching practices. Rebecca experienced a significant transformation in her teaching practices, evident by the high association of the *Teaching* and *Transformations* categories. In her first interview, Rebecca reported:

78   (0:33:56.4)   Rebecca   Like I teach my class totally opposite of the way that I was taught, um as a student and especially younger.

Rebecca further reported in her second interview on her transformed practices.

4   (0:00:37.4)   Rebecca   My classroom practices are drastically changed from this

program and I'm actually excited to share this.

From her responses, it seems that Rebecca considered one of her great transformations to be the changes she made in her teaching practices, and she attributed those transformations to her experiences in the professional development.

Rebecca's students were a top priority in her teaching, evident by the high frequency of the code "students" (37%) in the *Teaching* category. Rebecca's transformed perspectives on mathematics and learning necessitated a change in teaching practices, toward more learner-centered problem solving in order to keep her teaching practices aligned with her own experiences. Thus, Rebecca persevered over time to transform her practices because her transformed view on mathematics and learning led her to feel that changing her practices would provide important benefits to her students.

Some specific transformations in Rebecca's teaching practice were made evident in the *Teaching* category. "Telling" (22%), "allowing" (19%), and "authority" (19%) were all frequent codes within the *Teaching* category. Rebecca described her initial efforts to not show or "tell" her students how to solve problems before they worked on them. Despite her students' initial frustration, Rebecca remained committed to decrease "telling" as a teaching practice. Rebecca's decrease in "telling" behaviors was connected to a consequential increase in allowing students to exercise personal agency to develop their own ideas and take ownership for their own mathematical understanding. Rebecca's shift to a student-centered classroom was associated with a shift in authority. When Rebecca allowed her students to significantly join in the mathematical discourse of the classroom, she no longer was the sole mathematical authority but shared the authority

with all her students. Rebecca's classroom practices evolved concomitantly with her own

experiences as a learner of mathematics in the professional development classes.

Rebecca's responses in her exit survey provided additional details on her

transformed teaching practices. Rebecca's transformed practices focused on building

students' confidence and giving them the freedom to discover mathematical concepts.

She also engaged her students in problem solving open-response tasks.

Rebecca's Transformation in Confidence

A major transformation for Rebecca was in her confidence. In her first interview,

Rebecca described that although she still felt inadequate at times, she was able to become

more confident in her mathematical ability through the professional development.

78	(0:35:27.4)	Rebecca	And I think my confidence, although I don't, although I probably know that I'm one of the slowest thinkers in this class, I, my confidence in my math ability has heightened because of this class [the professional development].

The clips in the *Mathematics* category were frequently coded with "confidence" (30%).

As Rebecca's perspectives on mathematics transformed and enlarged to include sense-

making and problem-solving as important aspects of mathematical activity and as

Rebecca grew to consider that there are multiple ways of doing things mathematically,

she began to view herself as a capable mathematics learner. Her perspectives on

mathematics began to include the idea that mathematics is a creative activity in which she

and her students were all capable of engaging. As she said in her second interview, it was

through the *Placenticeras* task that Rebecca realized "I do not have to necessarily be a

college level mathematician to understand how math works" (line 16). Rebecca

transformed from holding the perspective that mathematics was for mathematicians only,

to gaining the perspective that mathematics is accessible for all eager learners. Parallel to her change in perspective was a transformation in confidence in herself as a mathematical learner.

In her first interview Rebecca discussed how her confidence in her mathematical understanding affected the transformation of her teaching practices over her second and third years of teaching (see Graven, 2004).

88	(0:38:19.2)	Rebecca	And then I, there were times [during my second year of teaching] where I wasn't confident enough in my own understanding of the math to where I would go back to the, you know, the easy way [making quotation marks in the air] of step by step. But this year I feel so much more confident.

Consistent with current literature, Rebecca needed to gain deep and flexible knowledge of mathematics to transform her teaching (Borko, 2004; Darling-Hammond, 2000; Simon & Schifter, 1991), but even further, she personally needed to gain confidence in order to fully commit to employing her transformed teaching practices. In her second interview, at the end of the professional development, Rebecca was able to report tremendous success with her new teaching practices. In the last year of the program Rebecca was able to create tasks for her students to engage in problem solving and collaboratively work as a class to build mathematical understanding. Rebecca's increased confidence was a critical component of her transformed teaching practices.

Rebecca's change to view mathematics as accessible for all also led her to allow her students more opportunities to engage in mathematics and strengthen their own confidence. In her second interview, Rebecca referenced this increased confidence her students experienced as she allowed them more opportunities to exercise personal agency as they engaged in problem solving.

4	(0:01:31.5)	Rebecca	They [my students], their confidence as a whole, academically, like I have kids who came in this year feeling academically like they've always had a hard time and math alone. Just that time with them and letting them have the freedom to think through their own ideas and not be told that it works one way and 'why can't you get that one way' and stuff, and that has really built their confidence and mine as a teacher.

Rebecca's increase in confidence sustained her in making difficult changes in her teaching practices, which afforded her students opportunities to increase their own confidence as well.

Clips from all Four Core Categories

Five clips were coded with all four of the core codes: "mathematics," "learning," "teaching practices," and "transformation." This data provides a more complete understanding of the transformations Rebecca experienced.

In the first interview, Rebecca described how her students liked her new teaching practices. While they came to enjoy mathematics, they did not have such positive affect at the beginning of the year. Rebecca described her students' initial disbelief when she would not show them how to solve problems before asking them to work on problems.

78	(0:34:13.9)	Rebecca	They [my students], they at first were like 'You've got to be kidding me. You have to show us how to do it first.'

Later in her first interview, Rebecca mentioned that she especially enjoyed implementing new teaching practices when she had the time to create lessons to her satisfaction.

90	(0:39:18.8)	Rebecca	Especially when I have the time to start creating lessons the way I know I want to teach it. That's cool.

In the second interview, Rebecca described her transformed teaching practices and her students' reactions.

| 4 | (0:02:03.5) | Rebecca | We just take a totally different route to it and we just throw problems out and go 'wow what do you think' and let them think, and some of them are so excited about what they discover, and that they've come to a problem in a totally different way but they have the right solution and they just think they're so cool and I think they're cool, and we're all cool. |

These three clips (lines 78, 90, 4) show that Rebecca began transforming her perspectives on teaching before she transformed her teaching practices, which is consistent with current research (Ball, 1996; Simon & Schifter, 1991; Zaslavsky & Leikin, 2004). Rebecca had a "way" she wanted to teach, which necessitated the creation and implementation of new lessons. Transforming her perspectives and practices led Rebecca to negotiate new didactical contracts (Brousseau, 1997) with her students and to expect students to problem solve. Rebecca persisted in allowing her students to be creative, approach problems from multiple viewpoints, and discover their own solutions, until she and her students found success. Her students came to respond with enthusiasm and excitement as they learned mathematics in a classroom that valued personal agency in problem solving. Rebecca's choice to persist with her transformed teaching practices, even in the face of initial resistance from her students, showed a deep commitment to new perspectives on mathematics and mathematics learning. Rebecca transformed to the perspective that important components of mathematics learning include approaching problems from different angles and allowing students to be creative. Rebecca also changed her conception of mathematics from a static collection of formulae to a creative, social, problem-solving process.

Later in the second interview, Rebecca discussed what she felt other teachers needed to do to begin transforming their own practices.

| 12 | (0:06:50.7) | Rebecca | And, um, I hope this doesn't sound pep rally-ish, but they [teachers] need to get fired up about how exciting the possibilities are when you approach math with a 'ok, we're not going to go down this one alley that I've planned, we're going to see what the kids can do and how they can create their own way to this problem.' |

Rebecca had transformed her perspectives on mathematics and learning and had also transformed her teaching practices. Her transformations had gotten her "fired up" and she wanted other teachers to see the exciting possibilities that she was seeing in her classroom. Instead of giving teachers all mathematical authority to control activities in the classroom, Rebecca advocated teachers allowing their students to approach mathematical problem solving creatively. In her exit survey, Rebecca also described her suggestions for helping other teachers transform. She described her vision for her role as numeracy coordinator as "a motivator and an example . . . by showing them what kind of success comes out of this approach [to teaching mathematics]." She said that she needed the other teachers to have "open minds" in order to help them engage in transformative experiences. Both Rebecca's response in the interview and in the survey show that Rebecca felt that teachers needed a successful example to watch, with open minds, to be able to follow that example and find the motivation to make difficult changes.

At the very end of the second interview, Rebecca was asked if there was anything she had not been asked but would like to share. She said:

| 20 | (0:12:52.5) | Rebecca | I hope they encourage many teachers to do something like this [professional development], because it has been… I thought I was just going to learn more math, not a whole way of teaching and thinking about it personally, and so I was like 'wow.' |

Rebecca had originally gotten involved in the professional development because she felt weak in her mathematical knowledge and wanted to learn more. She was

surprised to find that the experience led her to much deeper learning—actually transforming her perspectives on mathematics and her teaching practices. Significant in Rebecca's language is the contrast between "just going to learn more math" as compared to "a whole way of teaching and thinking about it." Rebecca believed she was "just" going to learn mathematics from the professional development, implying that by the end of the program she deemed learning mathematics as less significant than learning a "whole" new set of ideas about the subject and how to teach it. The implications of Rebecca's choice of language revealed that another transformation Rebecca experienced was a shift in values. She originally wanted to learn more mathematics, but later realized that merely learning mathematics was not enough to become the most effective mathematics teacher. Instead, she had to explore new conceptions of what mathematics really is, how it is learned, and how she could teach it to her students. Rebecca not only transformed her teaching practices and perspectives on mathematics, but she transformed her value system to value these new teaching practices and perceptions more highly than advanced mathematical knowledge alone. Rebecca's response also showed that she felt that these transformations came from her experiences in the professional development.

<div align="center">Code Frequencies</div>

I performed several searches for the frequency of various codes in the interview videos and in the *Placenticeras* task video. The results of these searches yielded important information that was not readily available from an analysis built from only the four core theoretical categories.

Interviews

Searching the interviews for code frequency brought forward one very interesting, seeming contradiction. "Reporting on success" (20%) and "noting inadequacy" (15%) were both frequent codes, but they appear to contradict one another. If Rebecca felt so successful about her experiences, then why would she frequently discuss her own inadequacy? I was surprised to find that there were even responses in her interviews where Rebecca would discuss a success and an inadequacy within the same sentence. The following (lines 78, 6, and 16) are a few examples of this paradox.

78	(0:35:27.4)	Rebecca	And I think my confidence, although I don't, although I probably know that I'm *one of the slowest thinkers* in this class [the professional development], I, *my confidence in my math ability has heightened* because of this class [professional development].

6	(0:03:49.4)	Rebecca	So it's fun; and *are we all geniuses [my students and I]? No*. But our *testing scores are looking really good*. Yeah, so that's a fun thing to actually see the difference.

16	(0:10:39.8)	Rebecca	I don't know. I just think if they [my students] see how math works 'cause I, I never ever did [taps table with her hands] and *now, I'm just like, oh wow, it all just comes together*. And it still *isn't a perfect puzzle piece*, of course, but it's fun.

Rebecca realized that she still had improvements to make in her teaching, her mathematical understanding, and in helping her students understand important mathematics. She could recognize her successes as well as those of her students, but she was not blind to the areas in which she needed to continue to improve. It turned out that her views were not contradictory, but that she had a healthy view of herself and her students as works-in-progress: having inadequacies but striving to improve and become

successful. Since professional development cannot provide a "recipe" for transforming teaching practices (Schifter, 1996, p. 496), such humility may be the best possible attitude for teachers leaving any professional development program—to realize one's progress and make plans for the areas one needs to improve.

Another aspect of the juxtaposition of "reporting on success" and "noting inadequacy" was that Rebecca felt more successful in some areas than in others. Specifically, Rebecca felt successful in changing her conceptions of mathematics and teaching and in implementing those changes, but she still felt inadequate in her mathematics knowledge. In her second interview Rebecca explained how she was so excited to share how successful her new teaching practices were because she felt like she was "not to that point mathematically" to share about the mathematical knowledge she was gaining. While Rebecca knew she had other successes she could report, she felt inadequate in her own knowledge of mathematics. She animatedly described the changes she had made in her perspectives on mathematics and the drastic improvements she had taken to transform her teaching practices, but when asked what one thing she would change about her mathematics teaching, Rebecca responded "That I still don't know enough . . . it drives me crazy that there are so many things that I could have known had I originally taken this approach in my thinking." Although Rebecca felt inadequate in her mathematical knowledge, she knew that she could "Learn more! Think more! Practice more!" Rebecca again displayed a healthy, humble, and appropriate perspective for a teacher leaving a professional development. Rebecca's inadequacies did not leave her paralyzed in self-doubt or fear because she knew she would leave the program and continue to learn and improve in the areas in which she felt weak.

Placenticeras Task

Examining the common behaviors engaged in during the *Placenticeras* task helped describe the particular conditions of this mathematical experience that was so transformative for Rebecca. Recall that Rebecca's work on the *Placenticeras* task was a significant experience for her to feel that she could "understand how math works" and how it "all comes together" (line 16). In their work on the *Placenticeras* task, Rebecca and her group members were able to engage in difficult mathematics and come to a solution that they felt was more compelling and complete than the typical solution of using an exponential function only. Here I describe the findings from the frequency searches of the *Placenticeras* task video clips, first from the *Placenticeras* task behavior codes and then from the Interview codes.

A search of the *Placenticeras* task clips for the frequency of *Placenticeras* task behavior codes revealed that over half of the coded clips (51%) involved collaboration and only 3% of the clips were coded "working on my own." Even though there were times when participants worked independently on the task, the majority of their work was collaborative. The collaborative nature of their work helped Rebecca see the task being approached from multiple viewpoints, which eventually became an important component of her transformed perspectives on mathematics. Collaboration was an integral part of the mathematics learning of the professional development in all the tasks, not just the *Placenticeras* task. However, the *Placenticeras* task gives a representative idea of how heavily the program relied on the collaborative interactions of participants to generate and develop mathematical ideas. The collaborative nature of the mathematical learning

was an important part of what made the professional development so transformative for Rebecca.

"Questioning" (27%), "explaining" (12%), "describing" (10%), "comparing" (10%), and "showing" (8%) were prominent collaborative behaviors in the *Placenticeras* task clips. It seems significant that "questioning" was so frequent, occurring more than twice as often as any of the other four behaviors. Nearly every instance of questioning during the *Placenticeras* task video clips involved participant-to-participant questions. Participants were able to bring together their varied ideas and approaches as they questioned one another on how, why, and what they were doing. While explaining, describing, comparing, and showing were important collaborative interactions for Rebecca and her group, their questioning played a more vital role in developing their eventual solution and yielded a greater contribution to their mathematical understanding. The centrality of questioning in participants' interactions aligns with previous research on student questioning (Walter, 2004).

A search of the *Placenticeras* task video clips for the Interview codes revealed that "prior experience" (6%) was the highest occurring Interview code. Rebecca and her collaborative group used their prior experiences to shape their individual approaches to the *Placenticeras* task. "Enthusiasm" (3%), "excited" (3%), and "enjoying" (2%) were also frequent codes in the *Placenticeras* task video clips. These three codes show the positive reactions that Rebecca and her group members gave to their work on the *Placenticeras* task. They especially displayed enthusiasm and excitement near the end of the task when they were able to see their ideas coming together to form a coherent solution.

"Figuring it out" (3%) and "making connections" (2%) were two other frequent codes that help describe the mathematical activity participants engaged in during the *Placenticeras* task. Rebecca and her group members were required to engage in problem solving without interventions from the instructor. Through individual, agentive approaches and thoughtful collaboration, the participants were able to make connections and eventually "figure out" a solution to the task.

CHAPTER 7: DISCUSSION AND IMPLICATIONS

I began my research looking for Rebecca's specific transformation*s* through the professional development and the transformative experiences that sustained, supported, and guided Rebecca in making such changes. Analysis of the *Transformations* category showed that Rebecca transformed her perspectives on mathematics, learning, and teaching, and especially her teaching practices. She also experienced a significant increase in her confidence as a teacher and a mathematics learner. She felt successful and enjoyed her personal transformations, and she noticed that her own changes afforded positive transformations for her students.

Analysis produced greater understanding of Rebecca's transformed perspectives on mathematics. She conveyed enthusiasm and confidence in the mathematical understanding she gained through the professional development. By the end of the program Rebecca was able to personally feel ownership for her mathematical understanding, in contrast to the beginning of the program when she felt that she never kept up in her mathematics classes enough to know what she was doing. Rather than viewing mathematics as a collection of rules and formulae, Rebecca's new perspectives on mathematics focused on building personal mathematical understanding, looking at mathematics from multiple viewpoints, making meaningful connections, exercising personal agency, and building on prior personal and collaborative experience. Rebecca saw mathematics not as a subject of rote memorization but as a creative, social endeavor and an exercise in personal agency.

Individual use of personal agency in approaching mathematical inquiry was an integral part of the mathematics learning of the professional development, which Rebecca

integrated into her own perspectives on mathematics, suggesting that Rebecca's

transformed perspectives on mathematics were based on her mathematical experiences in.

the program. For Rebecca, a transformative experience that prompted her to change her

perspectives on mathematics was engaging in learning mathematics in the professional

development setting where mathematics was viewed as a creative, social, problem-

solving activity with a special emphasis on the individual learner's use of personal

agency. Also important in facilitating a true learning experience for Rebecca was that the

mathematics content of the professional development was difficult and advanced beyond

elementary-level curriculum. Figure 23 shows mathematics learning, under the conditions

of difficult content, collaborative inquiry, and personal agency, as a transformative

experience that can sustain transformations in perspectives on mathematics. Although a

simple graphic here, Figure 23 becomes a building block that is incorporated into more

complex transformative experiences that are discussed later in this chapter.
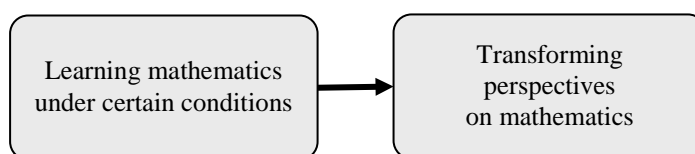
Figure 23. Learning mathematics sustains transforming perspectives on mathematics.

 Rebecca's experience might suggest that future professional development programs

should provide opportunities for participants to learn mathematics in ways that value

collaborative inquiry and that engage participants' personal agency in ways that can be

productive for the learner. Teachers can transform their perspectives on mathematics

when given opportunities to have transformative experiences by learning difficult

mathematics through collaborative problem solving and learner-centered generation of

knowledge with a focus on enacting personal agency.

Analysis provided greater detail for understanding Rebecca's transformed perspectives on mathematics learning. Her perspectives on learning were closely connected to her perspectives on mathematics and teaching. She often spoke of her students' learning, but also focused on her personal experiences in learning mathematics. Rebecca's perspectives on learning were closely tied to confidence, understanding, ownership, ability, and making connections. In contrast to her early self-reported sense of her capacities in doing mathematics, at the conclusion of the program she felt enthusiastic and successful in her efforts to learn mathematics and in her students' progress as well.

Rebecca's transformed perspectives on learning were based on her transformed view of mathematical activity. As she transformed her perspectives on mathematics to value problem solving, looking at mathematics from multiple viewpoints, and collaboration as important forms of mathematical activity, she changed to view learning mathematics as actually engaging in these activities. Thus, the transformative experience that directed Rebecca to transform her perspectives on learning was the very occurrence of transforming her perspectives on mathematics. The sequential nature of Rebecca's transformations implies that perhaps future professional development should focus on first giving teachers experiences to transform their views on mathematics before pushing for or expecting dramatic changes in perspectives on learning. Specifically, professional development programs could give teachers the opportunity to learn difficult mathematics in ways that value problem solving and personal agency. Figure 24 shows the transformative experiences that sustain transformations in perspectives on learning.

```
┌─────────────────┐      ┌─────────────────┐      ┌─────────────────┐
│    Learning     │      │  Transforming   │      │  Transforming   │
│   mathematics   │ ───► │  perspectives   │ ───► │  perspectives   │
│  under certain  │      │  on mathematics │      │   on learning   │
│   conditions    │      │ and mathematical│      │                 │
│                 │      │    activity     │      │                 │
└─────────────────┘      └─────────────────┘      └─────────────────┘
```
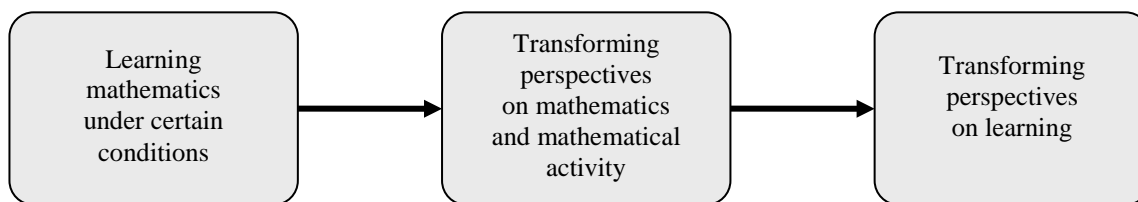
Figure 24. Learning mathematics sustains transforming perspectives on mathematical activity, which sustains transforming perspectives on learning.

Continued analysis presented greater detail in understanding Rebecca's transformed teaching practices. Rebecca showed a strong concern for student learning in her teaching practices. She was very cognizant of the effects her transformed practices were having on her students, and often reported on successes she and her students were experiencing as she persisted in changing and improving her teaching. Rebecca reported on her transformed practices with confidence and enthusiasm. Rebecca specifically discussed a decrease in "telling" and an increase in "allowing" in her teaching. Rebecca allowed her students to reason through mathematics problems instead of telling them step-by-step what to do. Allowing students to exercise agency to develop ownership for their own mathematical understanding led to changes in the mathematical authority in her classroom—Rebecca was no longer the sole authority because the authority was shared among all participants in the classroom. Other aspects of Rebecca's new practices included connecting mathematics to other subjects, problem solving, allowing students the freedom and agency to discover mathematical concepts, approaching problems from multiple viewpoints and collaboratively finding solutions, and reminding all students that they are capable of engaging in mathematical activity.

Rebecca transformed her teaching practices through a complex series of transformative events (see Figure 25). At an immediate level, she transformed her teaching practices through 1) transforming her perspectives on teaching and 2) gradually

implementing new practices in her classroom. Rebecca did not achieve such drastic changes in her teaching practices until after she had opportunities to try out new practices with her students over the three years of the professional development. Her concurrent classroom experience was vital in allowing her to align her actual practices with her transformed perspectives. But Rebecca's transformed perspectives on teaching can be traced back to another level of transformative experiences: 1) engaging with an effective pedagogical model and 2) transforming her perspectives on mathematics and learning. With Rebecca and the other participants as the learners, the instructor of the professional development modeled effective teaching practices of allowing student-to-student questioning and collaboration to generate the mathematics, inviting students to engage in problem solving, and allowing students' agentive choices to direct their own approaches to solutions. As Rebecca learned mathematics, she simultaneously interacted with the instructor which provided Rebecca a substantial pedagogical model to learn from and emulate. Although an effective pedagogical model was important, Rebecca's transformed perspectives on teaching were also based in her transformed perspectives on mathematics and learning. Rebecca's commitment to her new perspectives on teaching was motivated by her significantly-transformed perspectives on mathematics and how mathematics is learned. As was previously discussed, her transformed perspectives on learning and mathematics developed through the transformative experience of learning difficult mathematics in an environment focused on problem solving and agency. Thus, the mathematics learning of the professional development, as well as the effective teaching practices that were modeled, began a chain of transformative events that prompted

Rebecca to transform her teaching practices as she engaged in a recursive process of learning how to implement them in her classroom.
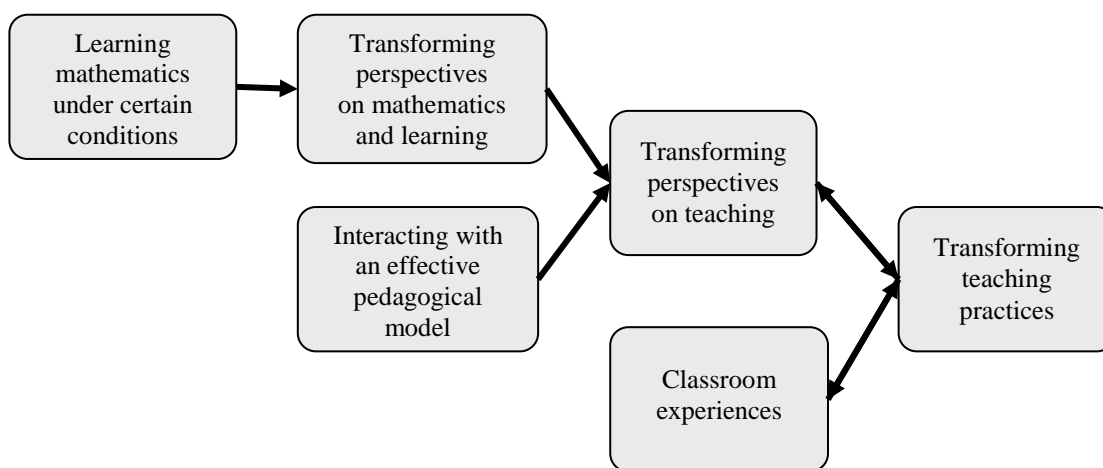
Figure 25. The complex process that can sustain teachers transforming teaching practices.

Rebecca's experience might suggest that professional developers hoping to help teachers transform their teaching practices should offer them opportunities to change their views on teaching as well as opportunities to practice and refine the teaching practices that align with those new views. To transform teachers' perspectives on teaching, professional developers could model effective teaching practices with participants as learners and also allow teachers to transform their perspectives on mathematics and learning. As was noted earlier, transforming perspectives on mathematics and learning may be achieved through allowing participants to engage in learning advanced mathematics under long-term conditions of conceptual challenge in collaborative problem solving and in concert with the expression or enactment of personal agency. Also, allowing participants to share and discuss adaptations of difficult tasks for their own students can facilitate participants being able to implement such practices in their classrooms more easily. For opportunities to attempt change and refinement in teaching practices, participants should be practicing teachers and should not

be taken out of their classrooms for long periods of time to participate in professional development. Perhaps more important would be to hold long-term, ongoing professional development programs during the regular school year and not during the summer, thus providing teachers ample opportunities to engage in a recursive back-and-forth process of transforming themselves as learners and transforming themselves as teachers.

Through Rebecca's experiences of learning mathematics in the professional development and in her efforts to implement transformed teaching practices, Rebecca experienced many successes but also acknowledged personal inadequacies. Through successful experiences Rebecca grew tremendously in her confidence as a teacher and mathematics learner. With confidence in and enthusiasm about her many successes, Rebecca desired to be an example and motivator for other teachers, desiring to become a leader in helping other teachers to transform their practices. Through awareness of her inadequacies, however, Rebecca developed an attitude of humility. She realized that she had many areas she still needed to improve. Rebecca's humility, coupled with confidence in her abilities, moved her to plan on continued learning and improvement after the professional development program. Professional development can place teachers on a path of career-long learning if teachers have sufficient opportunities to come to know their own successes and inadequacies. Figure 26 shows the transformative experiences that can lead to continued learning and a desire to be an example to others.
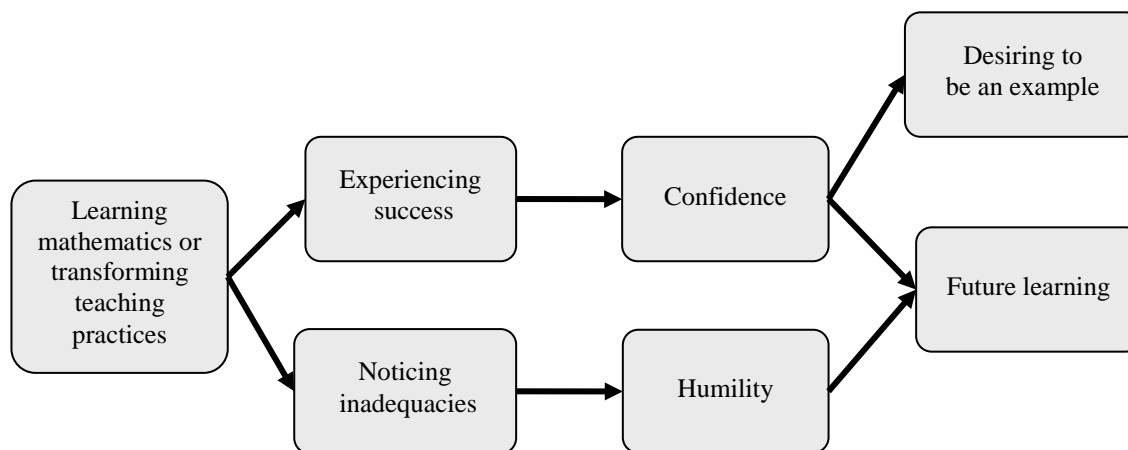
Figure 26. Experiences leading to becoming an example and continuing to learn.

The *Placenticeras* task was a transformative experience for Rebecca. It was the first task of the professional development where Rebecca believed that she could understand how mathematics works. The *Placenticeras* task did not occur until the end of the third semester of the professional development, after Rebecca had spent many hours learning mathematics and collaborating with other participants about mathematics teaching and learning issues. The fact that it took so long for Rebecca to find such a transformative task in the professional development experience supports the inference that short professional development programs may not be sufficient for helping teachers significantly transform their views on mathematics, teaching, and learning. My research on Rebecca is supported by and strengthens the current literature in mathematics education that asserts that effective professional development must be long-term (Ball, 1996; Borko, 2004; Cohen & Ball, 1990; Desimone, Porter, Garet, Yoon, & Birman, 2002; Schifter, 1996; Zaslavsky, & Leikin, 2004). However, some literature may define long-term as a few weeks or months, whereas this study shows that a year and a half of sustained professional development may be necessary for some teachers to reach transformative experiences.

Rebecca transformed her confidence, values, teaching practices, and her perspectives on mathematics, learning, and teaching. These transformations can be traced to her participation in the long-term professional development where she engaged in learning difficult mathematics by exercising personal agency and collaboratively problem solving with open-ended tasks. The instructor of the professional development was a superlative model of effective teaching practices by allowing students to generate sustained mathematical discussion directed toward building mathematical meaning, inviting students to exercise personal agency in problem solving, and expecting students to collaborate. Rebecca's recursive experiences in concurrently implementing new teaching practices in her own classroom also provided transformative experiences, with concomitant changes evolving over time for her students as well. Future professional development programs should grant access to long-term opportunities for teachers to engage as learners of difficult mathematics through agentive problem solving, to interact with appropriate learner-centered pedagogical models, and to simultaneously work through their transformations in their own classrooms. Such professional developments may provide transformative experiences that sustain teachers in making important transformations, endow them with the skills and determination to inspire other teachers, and encourage them to continue learning and transforming throughout their careers.

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APPENDIX A: INTERVIEW PROTOCOLS

First Interview

Name_____

- Have you taken calculus before?   (about 6 minutes)
If so, when?
How was it different from this semester's class?

    What have you learned this semester in calculus?

    How did you think about particular ideas to gain better understandings of concepts?

    What was interesting to you?

- How was your learning this semester different from previous courses? (about 4 minutes)

- How do calculus concepts that you learned in class relate to
    o Your students' mathematics content and learning?

    o Your classroom practices?

- How has this semester (past semesters) influenced
    o What you teach

    o How you teach

    o How you view mathematics learning

    o What mathematics your students do

- I've noticed over the past semester or two that something seems to have changed about how you view yourself as a participant in these classes. Could you help me understand more about that?

Second Interview

1. How have your classroom practices changed as a result of your participation in the professional development content courses?

2. How have your students benefited from your participation in the entire program? [How are your experiences in this math professional development translating into your classroom?]

3. As a numeracy coordinator, what do you envision for the future in the mathematics education of students in Provo School District?

4. Have you begun to work with teachers in your school? If so, please tell us about it. If not, what do you think could be a good beginning?

5. Looking back over all of our semesters together, all of the tasks we worked on, which particular task did you find most engaging, why? (mathematics explored?, kids did it...?, was easily adapted?, deep thinking, fun?)

6. Is there anything that you would like to say that I haven't asked you about?

APPENDIX B: COMPLETE LIST OF CODES

Interview Codes

aha moment	ability	all works together
allowing	authority	challenging
coming alive	confidence	controlling
creating	creative	disbelief
discovering	enjoying	enthusiasm
failed	fear	figuring it out
freedom	frustrating	hating math
learning	loving	making connections
mathematics	mathematics content	motivation
multiple viewpoints	noting inadequacy	ownership
parents	persisting	personally
Placenticeras	prior experience	reasoning
reporting on success	room for improvement	seeing how math works
social	students	teaching practices
telling	textbook	transformation
tried	understanding	

Placenticeras Task Codes

accuracy	adjacent	amplitude
angle	asserting	center
change	circle	coefficient
collaborating	comparing	constant difference
decreasing	degree	describing
difference	equation	explaining
exponent	exponential	frequency
graph	growth	hypotenuse
increasing	inverse	Jim's equation
linear	measurement/measuring	missing link
non-constant difference	not knowing	noticing patterns
opposite	opposite over hypotenuse	pendulum
period	playing around with it	plugging in
predicting	process	questioning
radian	radius	rate of change
ratio	ray	Rebecca speaking
relationship	seeing	showing
simple	sine	spiral
terminating	triangles	verbalizing
wave	working on my own	x
y		

APPENDIX C: REBECCA'S INITIAL SURVEY (RECREATED)

Name: Rebecca

Years of teaching experience: 1st year teacher

Grade levels taught: 5th grade

Schools taught at: Franklin Elementary

Degrees Earned: BS degree

Mathematics background:
     Math for elementary school teachers I and II

*I have had a miserable few days with the flu. I came into my class tonight to do this assignment because this is where I left my math folder. I should be better by tomorrow night!

Perspectives on Mathematics and Teaching

1. *List three necessary qualities of an excellent mathematics teacher.*

Experience based knowledge of math and how students best learn it, confidence, and creativity (mixed with a touch of humor).

2. *Which of the qualities you listed above, do you feel is your strongest? Please explain.*

Creativity and an idea of how my students best learn their math. I can see when they are engaged in the assignment or lesson. When it is entertaining and rewarding, they are always right with me. I can usually tell when they are lost and am able to make it exciting, most of the time.

3. *Which of the qualities you listed above, do you feel is your weakest? Please explain.*

Knowing math inside and out well enough to be able to break it down for my slower students, and explain the hows and whys to my higher students.

4. *What do you like most about mathematics? Please explain.*

The logic of it once it gets into your head. It is factual, not philosophical. There's always a solid solution or answer. It's challenging.

5. *What do you find least appealing about mathematics? Why?*

How I was taught math. I never felt I could keep up with all the procedures and algorithms and theorems enough to ever know what I was really doing. It seemed too complicated to ever really be enjoyable.

6. *Describe an optimum classroom environment for learning mathematics. Why is this an optimum? What would be the practices within this environment?*

A class that has pictures displayed of math found and used in the environment so that kids could see its application. A teacher that shared an enthusiasm for finding answers and solving problems so that the kids would catch fire with at least the desire finding a solution. Math would be integrated into all subjects, showing practical use and creating mathematical thinking. Basically, a scare-free environment where students could investigate answers and their own methods of solving various problems. It has to be fun, satisfy curiosity, and allow students to create rather than be constrained by methodology.

7. *How does your own classroom-learning environment at your school resemble the optimum you describe? Please elaborate.*

It doesn't in many ways. I follow the book, but I do try to use tons of manipulatives and ask questions that allow the students to create their own ways of looking and solving certain problems. I am constantly showing them why certain methods work. My higher kids love that, but my lower kids have no idea what to do with it sometimes. I am positive about math, but often discouraged with the confusion that some kids continuously experience.

8. *What mathematics do you most enjoy teaching to your students? Be specific and explain why you find these particular topics engaging?*

I wish I had enough experience to review the past and pick one out, but in my few months, I would have to say that graphing was awesome. There were so many ways to show them how we could record and organize information in real life. They enjoyed seeing how often we use information and organize it mathematically.

APPENDIX D: REBECCA'S EXIT SURVEY (RECREATED)

Name: Rebecca

Degrees earned:
Masters (SUU, July 2005)
BS degree (UVSC, 2002)

Certification level: Level II Math endorsement (July 2005)

Total number of years of teaching experience: 3

Grade levels taught: 5th

Schools taught at: Franklin

Mathematics courses taken:
Geometry/Algebra & Calc, trig (BYU, credits through SUU, 2003-2005)
College algebra & math in el. Ed I and II (UVSC, 2000-2002)

Other professional development in mathematics
NCTM conferences
   Predominant emphasis on: teaching strategies, classroom activities
   When: 2002-2003
    Usefulness: fairly

Perspectives on Mathematics and Teaching

1. *List three necessary qualities of an excellent mathematics teacher.*

Patience
Open mindedness (considering multiple ways of looking at a problem)
Personal prep. and study (KNOWLEDGE)

2. *Which of the qualities you listed above, do you feel is your strongest? Please explain.*

They are already listed according to my ability, 1 being where I am most capable. I can really help a student who is struggling.

3. *Which of the qualities you listed above, do you feel is your weakest? Please explain.*

Knowledge. I feel I have so many abilities to teach math well, but I've never had a true "math mind." I am starting to improve this, however.

4. *What do you like most about mathematics? Please explain.*

How it all works together! This class made me realize that math is NOT segmented, difficult to understand, technical theory. It's actually a web of patterns.

5. *What do you find least appealing about mathematics? Why?*

The way I was taught math [3 arrows, each one pointing to segmented, difficult to understand, technical theory] before this class. It was approached as a secret language that only "smart" people could speak.

6. *Describe an optimum classroom environment for learning mathematics. Why is this an optimum? What would be the practices within this environment?*

One that displays math pictures, practices math situations, and promotes math conversations as a typical, do-able part of daily life. Students won't be fearful of something that is common and approachable.

7. *How does your own classroom-learning environment at your school resemble the optimum you describe? Please elaborate.*

I've changed so many things! My students talk about mathematical similarities in their literacy, they problem solve constantly, and no one feels like it's wrong to get a wrong answer as long as it keeps directing them to the right one!

8. *What mathematics do you most enjoy teaching to your students? Be specific and explain why you find these particular topics engaging.*

Fractions, dec., and percents because it's been so easy to present it as something they encounter everyday. They have discovered so many examples of parts of wholes, and have even figured things out beyond their leveled curriculum.

9. *If you could change one things about your mathematics teaching,*

- *What would it be?:* That I still don't know enough

- *Why would you want to change it?:* Because it drives me crazy that there are so many things that I could have known had I originally taken this approach in my thinking.
- *How could you change it?:* Learn more! Think more! Practice more!

10. *Based on your experiences in this professional development program, what changes have occurred:*

- *In your view of mathematics?* Omigosh! Math is my favorite subject to teacher which was originally my most dreaded subject. The students love it! There is so much to be discovered!
- *In your teaching of mathematics?* I take a confidence builder approach that reminds each student that they are completely capable to learn math. Giving them the freedom and time to think through problems and discover has made them more confident.

11. *How do you see your role as numeracy coordinator unfolding? For example, your responsibilities, activities, objectives, and influences with teachers in your own school or district-wide, etc.*

A motivator and an example. I think we'll start out small and build by showing them what kind of success comes out of this approach. I can help prepare tasks, open up new ideas, and motivate participation.

12. *What challenges do you anticipate in fulfilling your role as numeracy coordinator?*

Not many. I think it will be a lot of work and I'll have to really stay on top of things, but once people start to see results, they'll come to appreciate these approaches and use them.

13. *Identify some of the essential resources you need in order to achieve your objectives as a numeracy coordinator in your school, or district-wide.*

Filming
Instruction time
Open minds